Airfoil sensitivity analysis to establish efficient angles of attack and velocity: Towards a more sustainable wing design

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Abstract Efficient aerodynamics around the airfoil can reduce the financial burden on the aviation industry, thereby aiding in sustainability. This study aimed at performing sensitivity analysis on the NACA0012 airfoil design to identify optimum mesh size, angle of attack (AOA) and velocity for best lift to drag ratio (L/D). Various machine learning (ML) algorithms were subsequently trained to predict L/D using the velocity and AOA. Our results showed that a mesh size of 225 divisions per unit, AOA of 2 degrees, provided the highest L/D. Velocity did not impact the L/D too much. Among ML algorithms, random forest had the highest accuracy in predicting L/D with a RMSE of 0.68. These results provide support to improvements to the airfoil aerodynamics that could potentially contribute positively towards the sustainability efforts of the aviation industry.

Key Words Airfoil, Sensitivity Analysis, Aviation Industry, Sustainability, Machine Learning (ML), Lift to Drag Ratio (L/D).

Introduction

The aviation industry has emerged as a critical pillar for transportation of goods & services, economic development and globalism. Since its inception in 1903 during the Wright Brothers' era, it has branched out into commercial, freighter and military operations. This has led to countries, cultures and economies being connected, enabling people and goods to travel from source to destination in less than 24 hours (International Civil Aviation Organization Agency). However, as demand increases, the aviation industry will need to expand constantly to meet these demands. In fact, Airbus has estimated that the number of flights will have doubled in the next 20 years (Topham). While travel costs for passengers may decrease, more resources and energy will have to be allocated to the manufacturing process. High travel rates will also increase fuel demand and cost and, therefore, the aviation industry's contribution to carbon dioxide emissions.

What has often been overlooked but is now gaining attention is the need for sustainability within the aviation sector: the sector's rapid growth has positioned it as one of the fastest-growing contributors to greenhouse gas emissions (Overton). Aviation currently has a 2.5% stake in global CO_2 emissions, and with a two-fold increase in air travel over the next two decades, the industry's footprint will undoubtedly grow larger (Ritchie). In response to growing concerns on climate change, the industry plans on reducing its carbon footprint by investing in optimizing flight plants, making remarkable progress in introducing sustainable aviation fuel (SAF) as well as modern aircraft designs with fuel-efficient systems (International Air Transport Association).

Design and aerodynamics around the airfoil plays an important role in the design of efficient flight wings, resulting in aircrafts, enabling more sustainable flight designs. A close attention is paid in improving the flow of air around an airfoil, which results in a low drag when compared to turbulent flow. Laminar flow is critical in reducing drag which, in turn, is essential for fuel efficiency, range and performance. This results in lower operational costs, allowing planes to fly longer distances with the same amount of fuel (GlobeAir AG). Ensuring efficient airfoils can be accomplished by designing wings with smooth contours and minimal surface roughness (GlobeAir AG).

To assess the aerodynamics around an airfoil, lift and drag are two quantities that are commonly evaluated. Lift quantifies the upward force applied on the airfoil due to the pressure difference, and drag quantifies the frictional forces on the surface of the airfoil (SIMSCALE). Lift-to-drag ratio (L/D) provides the proportional lift force normalized by the drag force. A high L/D ratio indicates that an aircraft generates high lift while minimizing the amount of drag, directly resulting in high fuel efficiency (Hall). Under cruise conditions, lift equals the plane's weight, so a high L/D ratio allows for larger payloads without increasing the amount of thrust. As thrust is generated from burning fuel in the jet engine, a reduction in the amount of thrust needed means less fuel is burned thus contributing less to greenhouse gas emissions (Hall). The airfoil shape, angle of attack and velocity of the airplane are responsible for determining the lift and drag produced by the wing. By changing the shape of the wing and the air conditions, aircrafts may achieve higher L/D ratios, improving performance and fuel efficiency over its lifespan. optimizations were used in this study as an indication of aerodynamic efficiency of an airplane.

To achieve higher efficiency, this study critically examines the design of a wing's airfoil through a series of sensitivity analysis including meshing, angles of attack, and velocities. Simulations were performed on an airfoil at different mesh sizes, angles of attack and velocities, with lift and drag coefficients quantified to determine L/D.Through this analysis, highest L/D ratios was identified to evaluate the most efficient conditions required for sustainably efficient flights.

Methodology

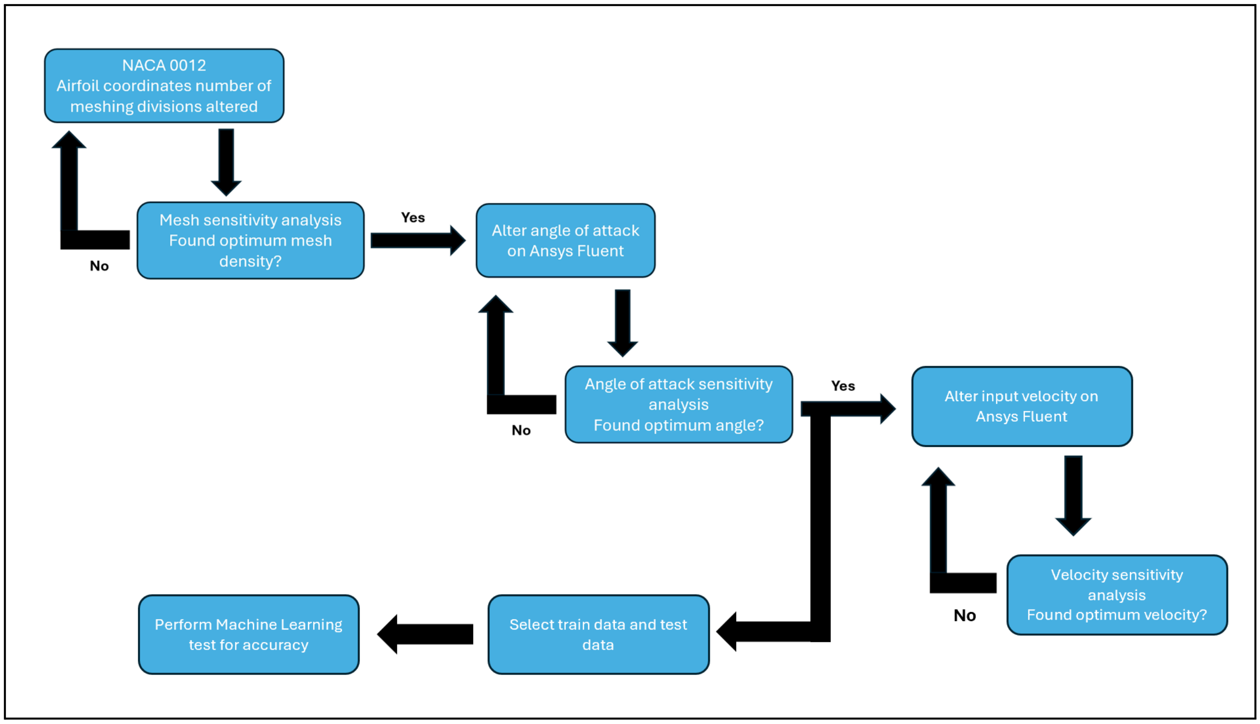

Figure 1. Flowchart to demonstrate methodology followed during investigation.

Figure 1 details the consecutive steps followed through this study - it details the use of optimal mesh values for angle of attack sensitivity analysis and the optimal angle of attack value for velocity sensitivity analysis.

Airfoil design and meshing

The NACA 0012 airfoil profile was chosen for this study due to its symmetrical design and aerodynamic properties. (Rumsey) A series of coordinates were used to generate the lower and upper sections of the airfoil. These coordinates were imported in Ansys DesignModeler (ANSYS Canonsburg, PA, USA) where they were connected through a series of vectors to build a two-dimensional framework. A C-Mesh domain was formed to efficiently capture the flow of air around the airfoil. The domain was constructed by sketching a semicircular arc upstream of the airfoil and a rectangle which extends towards the region downstream. The airfoil was centered in the meshing domain and areas around were refined near the airfoil's surface, mostly around leading and trailing edges, to capture the boundary layer results accurately.

Solver and system details

The computational fluid dynamic (CFD)simulations for this study were conducted on Ansys Fluent using a steady-state solver. A density-based solver with second-order upwind discretization was used to solve the flow-governing Navier-Stokes' equations. The fluid density was set to 1 kg m^{-3} to represent properties of air.

To simulate the incoming airflow, the inlet boundary was defaulted to a uniform velocity input. The outer at the downstream region end of the rectangular domain was defaulted to a pressure outlet, thus allowing air to flow out of the system. A velocity-based inlet was used to alter the angle of attack and velocity of the airflow. The airfoil surface was defined as a no-slip wall boundary, ensuring that the velocity at the surface is zero.

All simulations were performed using an Asus A15 laptop equipped with an AMD RYZEN 7 4800H processor and NVIDIA GeForce RTX 2060 graphics card (ASUS). The system features up to 32 GB of 3200 MHz DDR4 RAM and a 1 TB NVME PCIE SSD, operating on Windows 11 Pro. The FA506IH model of the A15 was used with processor specifications of 6/core12-thread, 11 MB cache and 4.0 GHz max boost. The run-time for each simulation modeled on this system varied between 46.8 seconds to 17.3 minutes.

Lift and drag calculations

Figure 2 demonstrates the forces acting around an airplane's wing's airfoil.

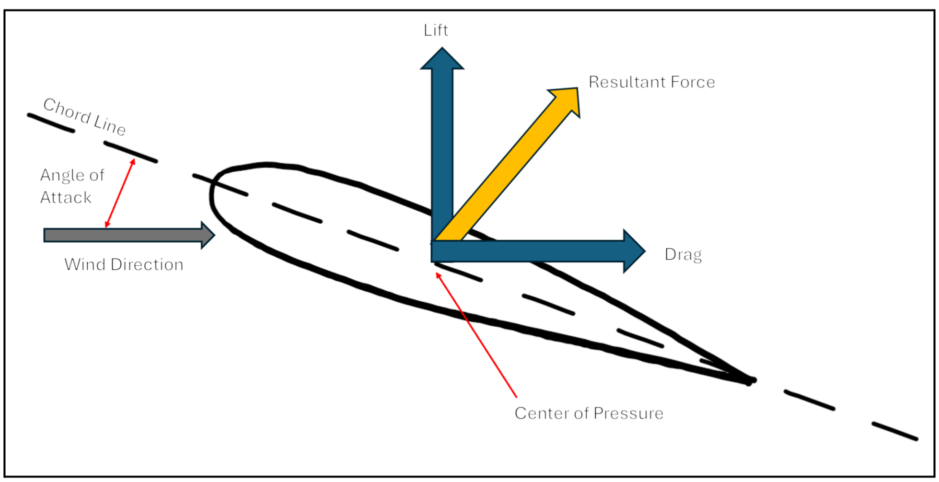

Figure 2. Free-body diagram of an airfoil

The lift force is the component of the total force vector that works at the center of pressure of an object; it is perpendicular to incoming flow (SIMSCALE). Drag is a rearward, retarding force caused by the disruption of airflow by the wing, fuselage, rotor and other protruding objects on the body of the aircraft (FAA).

Mesh sensitivity analysis

Mesh sensitivity analysis was performed to identify mesh-independent solutions for the airfoil to perform subsequent analysis. A mapped face control system was utilized to model the meshing system in Ansys mesher. This helps in reducing errors related to unstructured elements and provides better control over element distribution. A C-mesh domain was constructed with bias towards the airfoil to capture the boundary layer effectively.

The number of divisions on the meshing model was chosen as the independent variable for the sensitivity analysis - to calculate the effectiveness of each subsequent model, the lift-drag ratio was calculated. Values of 25, 50, 75, 100, 125, 150, 200, 225 and 250 divisions were chosen. The lift and drag coefficients were determined computationally at an x and y velocity of 1 ms^{-1}.

Angle of attack analysis

The Angle of Attack (AOA) is the angle at which the chord of an airplane's wing meets the relative wind; the chord is a straight line from the leading edge to the trailing edge (FAA).

The AOAs were altered through use of Ansys Fluent; a series of trigonometric ratios were solved to determine the x and y components of the intake velocity. The subsequent lift-to drag ratio is calculated.

Velocity analysis

The velocity intake can be split into two-dimensions to model the motion of air over the airfoil: the x-component and y-component of velocity. The x-component represents the horizontal motion of air against the airfoil structure, while the y-component represents the vertical motion of air. These components can be found through the sine and cosine trigonometric ratios. Values for the optimal AOA and mesh structure were used to calculate the velocity components.

Values above and below the average takeoff speed for an airplane (~ 240-285 kmh^{-1}) were chosen as part of the sensitivity analyses. Once again, the lift-to-drag ratio was determined and analyzed to find the optimum take-off speed for an airplane with NACA 0012.

Machine Learning Analysis

To generate models to efficiently predict L/D ratio given the angle of attack and velocity values, , machine learning (ML) algorithms were developed based on the sensitivity analysis results. This predictive model was trained to estimate lift-to-drag ratios for users unfamiliar with CFD modeling or lack the necessary system specifications to run the simulations. The ML models were trained on Python's NumPy, pandas and scikit-learn libraries (Python). Data sets in the form of a text file were imported and read into the program where it was split into test and train data in a split of 20%. Calculations were run on the train-data to determine the missing test-data values. The predicted-data was compared to the train-data to determine how close or far the calculated values are from the actual ones. Ultimately, each regression method was compared to determine the most accurate one by evaluating which method had the lowest mean squared error and the highest R^2 value.

Results

Mesh sensitivity analysis

A series of simulations were conducted on mesh edge sizes ranging from 25 to 275, and the lift-to-drag ratios were calculated. Airfoils with edge sizes at the lower end showed a difference of more than 39% in lift-to-drag ratios, while those at the higher end exhibited a difference of <5%. Table 1 presents the lift-to-drag values for edge sizing in increments of 25 divisions.

Edge Sizing (number of divisions)	Lift:Drag
25	34.22
50	86.46
75	143.09
100	92.68
150	247.52
200	271.60
225	272.79
250	288.03
275	306.86

Table 1. Edge sizing versus Lift:Drag, in increments of 25 divisions.

The smallest difference between consecutive simulations was observed at an edge size of 225, with less than a 0.5% change in value. As a result, airfoils with an edge size of 225 were considered the most reliable for performing subsequent analysis.

Figure 3 displays a plot of the lift-to-drag ratio against edge sizing (number of divisions), with a curve of best fit applied to the data. An exponential fit was selected due to the decreasing rate of change observed towards the end of the plot. Both Table 1 and Figure 3 clearly highlight the point (100, 92.67) as an outlier, as it deviates from the upward trend represented by the exponential curve.

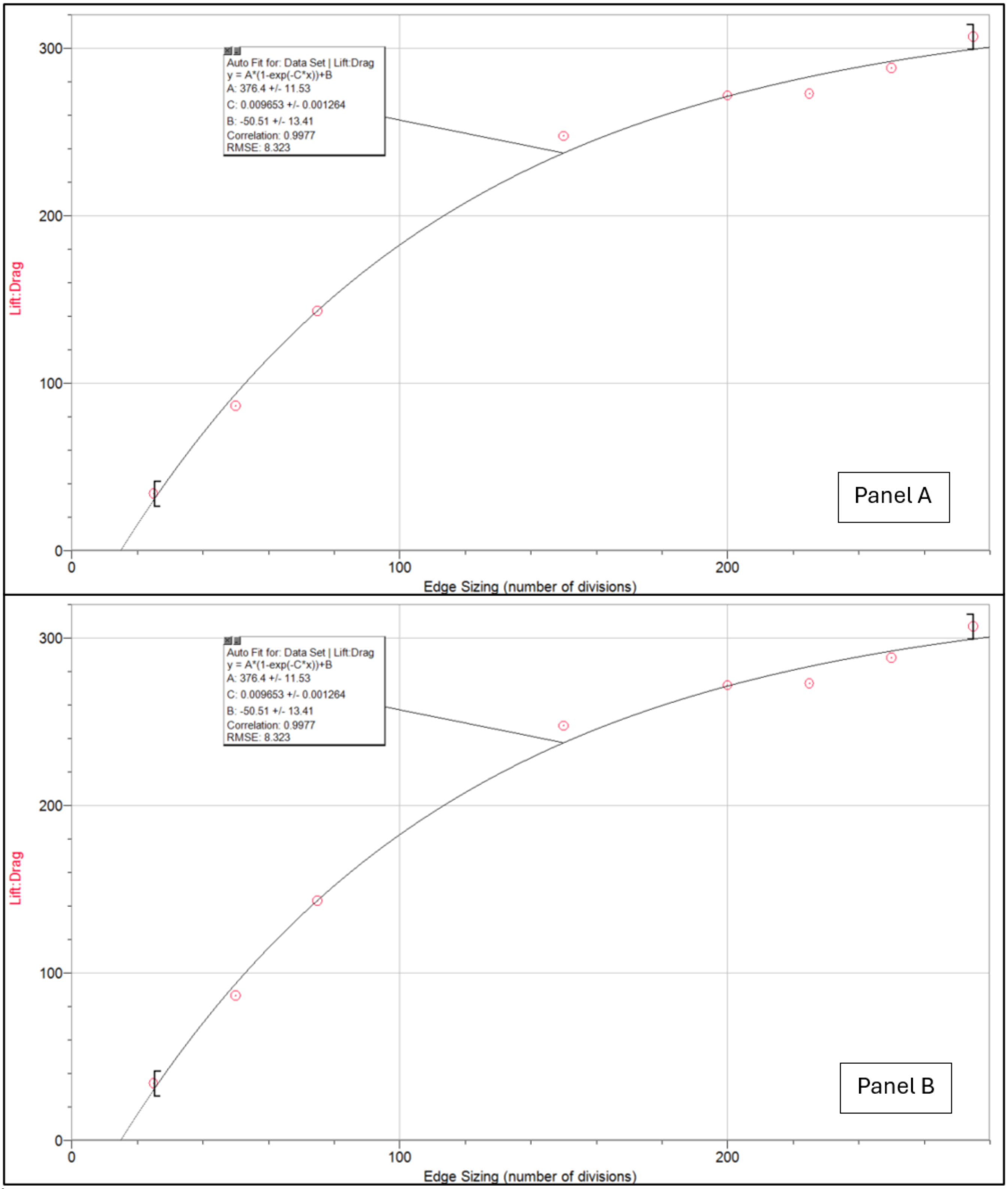

Figure 3. Plot of Lift:Drag against Edge Sizing, with Panel B filtering anomalies.

A separate curve was plotted in Panel B, excluding the outlier at 100 edge sizing, to provide a more accurate representation of the best-fit curve. This curve aligns with more data points compared to Panel A, confirming (100, 92.67) as an anomaly. Additionally, the correlation between values is significantly stronger (0.9977 vs. 0.9656), and the root mean square error (RMSE) is much lower (8.323 vs. 31.25). As a result, the equation of the curve for further calculations can be approximated as follows:

$$y = 376.4(1 - exp(-0.009653x)) - 50.51$$

Figure 4 illustrates the structured mesh applied to the airfoil surface, divided into 225 sections along the chord length to ensure accurate aerodynamic analysis. This fine mesh consists of 13,377 individual elements, carefully distributed to capture both the surface flow details and the boundary layer behavior. The mesh refinement around the leading and trailing edges of the airfoil allows for better resolution of high-gradient regions, particularly in areas prone to separation and vortex formation.

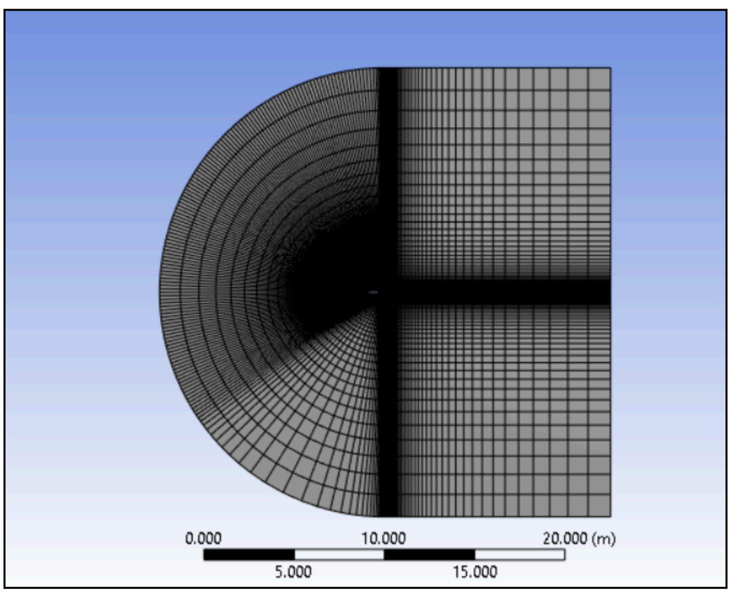

Figure 4. Mesh design at 225 divisions.

Angle of Attack Analysis

As identified in the mesh sensitivity analysis, 225 divisions were determined to be the optimal edge sizing factor for running further simulations. The angle of attack was varied in 2 degree intervals to establish its relationship with the lift-to-drag ratio. Smaller intervals were used initially to gain a clearer understanding of how the relationship behaves at lower values. The results from these angle of attack simulations are presented in Table 2.

Angle of Attack (°)	Lift:Drag
0	-0.81
0.5	251.40
1	408.93
2	505.55
3	450.40
4	386.85
6	272.79
8	207.51
10	159.20
12	127.73
14	106.07
16	83.13
18	79.40

Table 2. Angle of Attack versus Lift:drag, in increments of 2°**.**

Figure 5 illustrates the relationship between the lift-to-drag ratio and the AOA (in degrees) through a point-to-point plot. The graph shows a steep upward trend, peaking at (2, 505). No significant outliers were observed throughout the data.

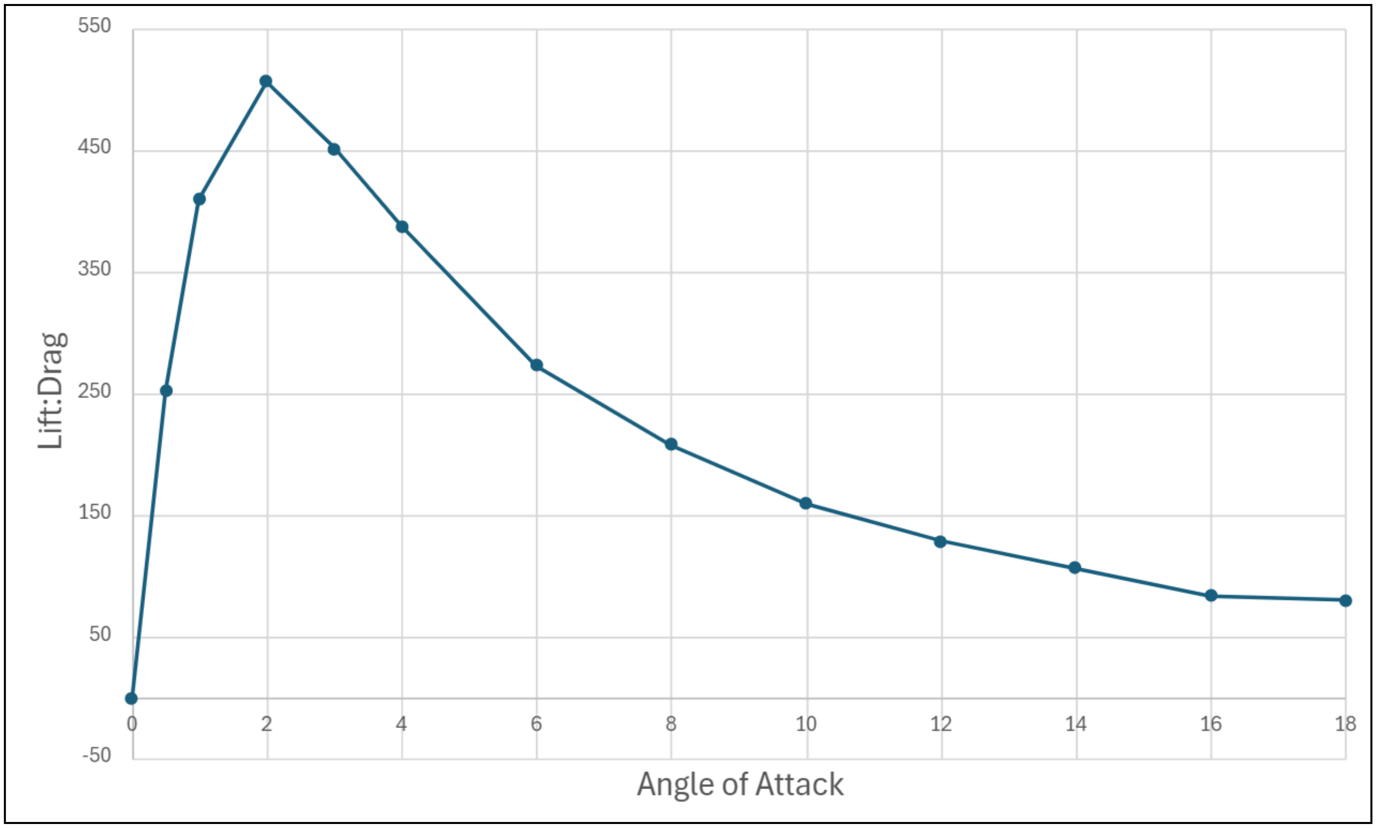

Figure 5. Lift-to-drag ratio plotted against AOA

As the angle of attack increases from 0° to about 3°, the lift-to-drag ratio rapidly increases, peaking at a value of around 450. This indicates that at these low angles of attack, the airfoil is operating efficiently, generating high lift with relatively low drag. After reaching the peak at 2°, the lift-to-drag ratio decreases sharply as the angle of attack continues to increase, suggesting that the drag becomes more significant compared to lift. Beyond 6°, the lift-to-drag ratio continues to decline steadily at a slower rate - the angle of attack further leads to a diminishing lift generation, as drag further increases. At the far end, the curve approaches a horizontal asymptote at a lift-to-drag at around 79 (the graph plateaus).

It is important to note that Table 2 and Figure 5 show anything less than an angle of attack of 0.5° will result in a negative value for the lift-to-drag ratio. Further, approximating a curve-fit for the plot above, a gaussian fit is the most suitable option for the relationship. Due to the high efficiencies reported at 1-4° angles of attack, these values will be used when conducting the velocity sensitivity analyses.

Velocity Analysis

To conduct this sensitivity analysis, the 225 edge sizing, coupled with the 1-4° angle of attack values, were used to determine the velocity for highest efficiency. Table 3 displays the results from the analysis, with velocities of 1-100 kmh^{-1} chosen. This allows comparison between taxi, cruise and take-off speeds.

Angle of Attack (°)	Velocities (ms^{-1})	L:D
1	1	408.93
	25	408.53
	50	408.55
	75	408.56
	100	408.54
2	1	505.55
	25	505.53
	50	505.55
	75	505.57
	100	505.55
3	1	450.40
	25	450.37
	50	450.28
	75	450.31
	100	450.28
4	1	386.85
	25	387.02
	50	387.02
	75	387.02
	100	387.02

Table 3. Velocity sensitivity analysis results with angle of attack consideration.

As can be seen from the above table, the values of lift-to-drag ratios show very little variation as the speed of the air passing across the airfoil cross-section. Despite this, there appears to exist a negative correlation between the velocity and lift-to-drag values. When comparing the values for lift-to-drag with the results achieved in the AOA sensitivity analysis (Table 2), the values seem to match up at the corresponding AOA (regardless of velocity).

As a precaution to ensure no significant relationship between the two variables, a Pearson's Correlation Coefficient test was conducted on an airfoil with AOA 2°. An r^2 value of 0.146 was noted, suggesting a very weak, if not, no correlation between velocity and its resulting lift-to-drag.

Machine Learning comparisons

As an extension to this research, different machine learning programs were used to generate relevant estimates for the lift-to-drag ratio for any angle of attack value provided. Table 4 presents the results for each algorithm based on their level of accuracy (r^2 value and MSE) in generating estimates for the testing angle of attack data.

Machine Learning Algorithm	Mean Squared Error	R^2 Score
Linear Regression	4.27	0.96
Ridge Regression	86.03	0.29
Lasso Regression	89.31	0.26
Decision Tree	3.33	0.97
Random Forest	0.68	0.99
Support Vector Regression (SVR)	42.28	0.65

Table 4. Machine Learning algorithm comparison by standards of accuracy.

According to the results, the random forest algorithm appears to be the most efficient and accurate form of determining lift-to-drag ratios for their relevant angle of attack values. This algorithm is closely followed by the decision tree and linear regression methods. However, the ridge and lasso regression algorithms appear to be the least accurate in the context of this study. Support Vector Regression is placed in the middle of the available methods used.

Figure 6 is a bar chart of 1/(ABS(log(Mean SquaredError))) against the method used to demonstrate the trend in a visual manner. A log scale was used to ensure all values fit the scale of the graph, while absolute values were used to nullify the negative sign on certain values with MSE less than 1.

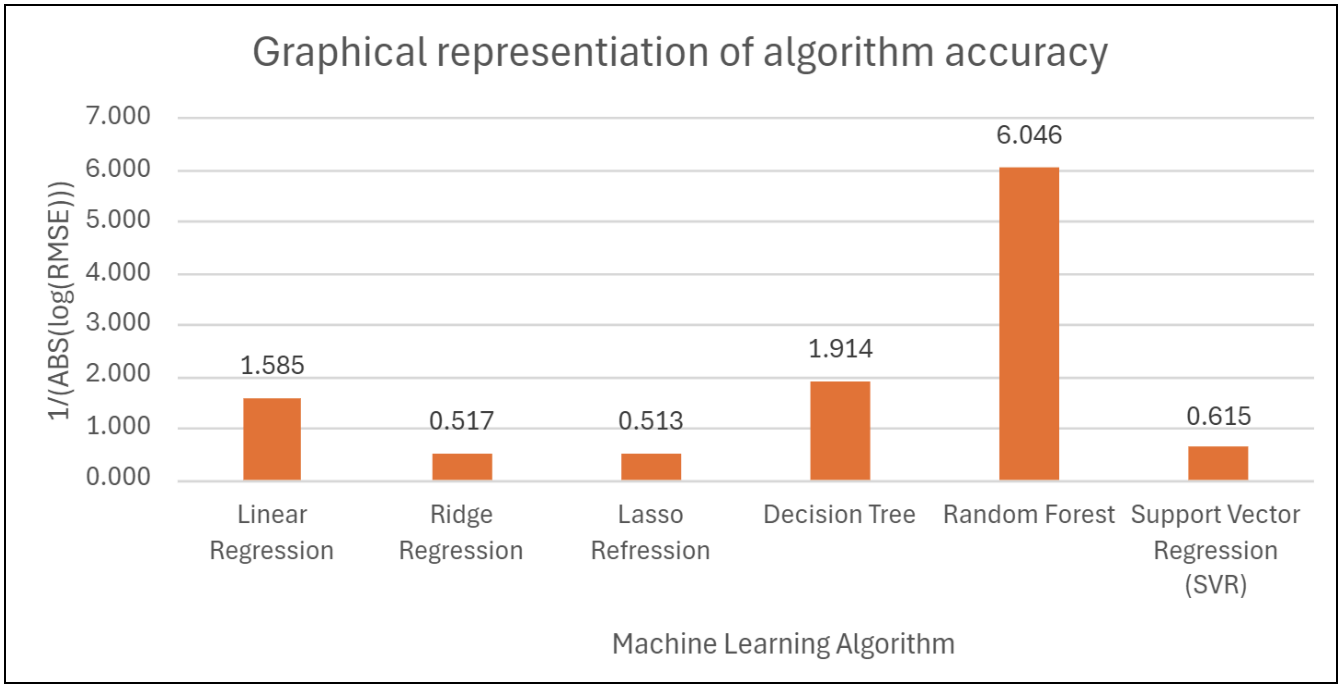

Figure 6. Graphical representation of algorithm accuracy.

Through this, the ranking of machine learning algorithms based on accuracy is clearly determined with random forest presenting the most valid results.

Discussion and Conclusions

This study analyzed the effects of three independent variables (meshing parameters, AOA and incoming air velocity) on airfoil efficiency, to optimize the lift-to-drag ratio. Our results show how adjustments to meshing parameters and AOA influenced the aerodynamic performance of the airfoil, providing valuable insights into optimizing the airfoil design for improved efficiency. We also found that the incoming air velocities had minimal contributions to the measure of efficiency, with lift-to-drag ratios remaining static irrespective of the velocity. In terms of the mesh sensitivity analysis, calculations pointed out that 225 was the most consistent and accurate value for obtaining lift-to-drag values. Hence, it was used to find the relation between the other two independent variables and aerodynamic efficiency. In the angle of attack sensitivity analysis, calculations suggested a maximum point at 2°. Finally, on ML based trained models for L/D prediction, we found that the Random Forest was the most efficient and accurate method for calculating lift-to-drag ratios with corresponding angle of attack values.

The AOA analysis results agree with previous studies. At low AOAs, the airflow over the top of the wing produces lift with a relatively small amount of drag (FAA). An increase in the AOA results in an increase in both the lift and induced drag, leading to a lower lift-to-drag ratio (skybrary). The lift coefficient increases linearly until a certain angle of attack (~ 17-18°), then variation becomes non-linear and the lift coefficient approaches a maximum (Cavcar et Al.). Similar trends with Cavcar et Al.'s study have been recorded as part of the AOA sensitivity analysis (Figure 7).

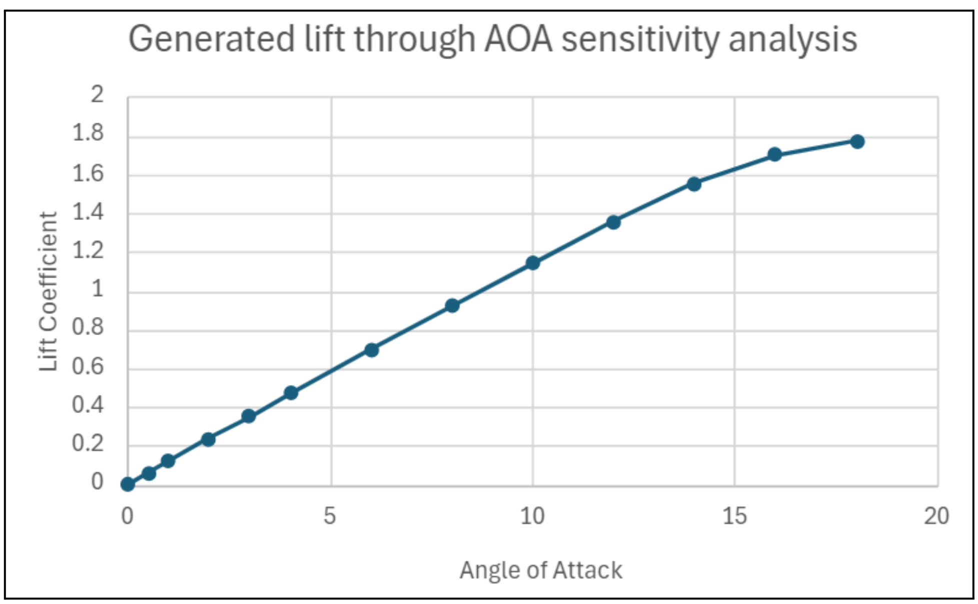

Figure 7. Calculated lift coefficients from AOA sensitivity analysis.

Higher AOA values are typically associated with airfoil shapes that influence flow separation. Therefore, higher drag values are estimated at larger AOA (Figure 8). Since the lift curve approaches an asymptote, drag overpowers the system inducing a stall; therefore, a suitable range within the region of linearity (0° - 20°) was chosen.

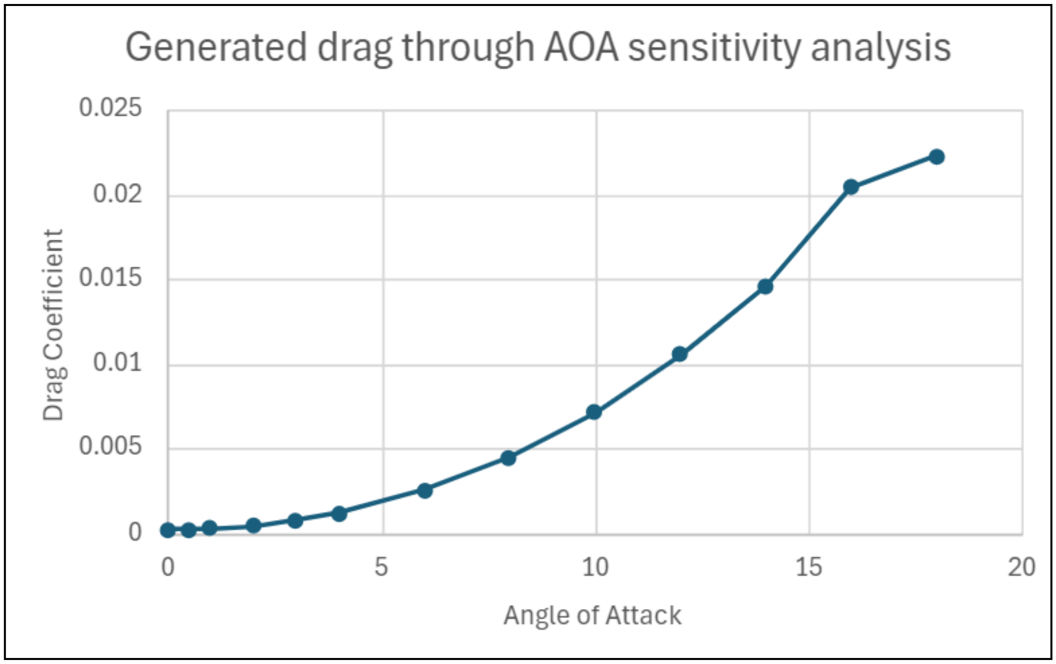

Figure 8. Calculated drag coefficients from AOA sensitivity analysis.

In the velocity sensitivity analysis, no significant changes in the lift-to-drag ratio were observed with increasing velocity. The only factor that appeared to affect the lift-to-drag ratio was the angle of attack (AOA), within the range of 1° to 4°. From Newton's second law of motion, the aerodynamic forces which act on the airfoil (lift and drag) are directly related to the change in momentum of the fluid with respect to time (NASA). Fluid momentum can be represented as the product of the mass and velocity of the fluid. Therefore, an increase in the velocity of the air will directly affect all the aerodynamic forces acting on the body. A series of derivations conducted by NASA revealed that both lift and drag depend on the square of the velocity. Therefore, when applying the lift-to-drag ratio, the velocity of the incoming air is expected to have no effect. The increase in velocity will account for equal increases in drag and lift, resulting in no overall change in the applied ratio.

For prediction L/D ratio, we employed multiple ML algorithms using the library scikit-learn.Linear regression models the relationship between a dependent variable and one or more independent variables by fitting a linear equation to the data, minimizing the mean squared error between predicted and actual values (scikit-learn). Ridge regression incorporates a regularization term that penalizes large coefficients, which helps in reducing overfitting and improving generalization (scikit-learn). Lasso regression modifies linear regression with an absolute value penalty, shrinking some coefficients to zero and performing feature selection (scikit-learn). Decision trees create a model by splitting data into subsets based on feature values, constructing a tree structure that helps in decision-making by partitioning the data to maximize the separation of target outcomes (scikit-learn). Random forests use a method of aggregating predictions from multiple decision trees, which enhances accuracy and mitigates overfitting by averaging the outputs of several trees trained on random data subsets (scikit-learn). Support vector regression seeks to approximate the data with a function that has a margin of tolerance, maximizing the margin between data points and the predicted function while allowing some deviations within this margin (scikit-learn).

As demonstrated in Figure 6, Random Forest was found to be the superior machine learning algorithm in terms of accuracy. According to Donges, Random Forest adds additional randomness to the model, while growing the trees. The algorithm searches for the best feature among a random subset of features, instead of focusing on the most important feature while splitting nodes. This could result in a wide diversity that results in a more accurate model (Donges).

In conclusion, this study critically examines the effect aerodynamic properties of fluid-velocity and angle of attack, as well as the mesh density, affects the lift-to-drag ratio of a NACA 0012 airfoil model. Generally, a positive relationship was established between the mesh density and lift-to-drag values. In terms of angles of attack, the ratio reaches a maximum at around 2° and decreases until it reaches a horizontal asymptote. The fluid velocity had an infinitesimally small impact on the lift-to-drag ratio, when compared to the angle of attack variations. Finally, through the machine learning comparisons, the Random Forest algorithm was determined to be the most accurate means of generating lift-to-drag values from any angle of attack value (0°-20°).

There is now an overwhelming amount of compelling evidence which suggests CO_2 is responsible for human-induced climate change, leading to exacerbated global warming conditions. Global warming is responsible for diurnal temperature increases, melting of polar ice caps, rising sea levels, and more dangerous coastal storms. It is, thus, important to tackle global warming by addressing key contributors to global CO_2 emissions; sustainability is getting ever more important in today's world, let alone the aviation industry. In 2022, the ICAO has agreed to initiate the net-zero CO_2 emissions by 2050 (Scott). In fact, multinational manufacturers, such as Airbus, have pledged to reach the net-zero goal by 2035 through the use of renewables and fuel-efficient engines.

Data availability

The processed data including ML code is available upon suitable request.

Authorship and acknowledgements

Aarin Rakesh: Writing - Conceptualization, Methodology, Lead Author. **Nikhil Paliwal:** Writing - Conceptualization, Supervision, Review & Editing.

References

1. ASUS. “ASUS TUF Gaming A15 | Laptops for Gaming | ASUS Global.” *ASUS Global*, 2024, www.asus.com/Laptops/For-Gaming/TUF-Gaming/ASUS-TUF-Gaming-A15/.

2. Cavcar, Mustafa. “Stall Speed.” *Anadolu University, School of Civil Aviation, Turkey*, 2004.

3. Corp, Spatial. “Meshing Definition | What Is Mesh Generation | Spatial.” *Www.spatial.com*, 2024, www.spatial.com/resources/glossary/what-is-meshing.

4. Donges, Niklas. “Random Forest: A Complete Guide for Machine Learning.” *Built In*, 22 July 2021, builtin.com/data-science/random-forest-algorithm.

5. FAA. *Forces Acting on the Aircraft Aerodynamics of Flight*. 2024.

6. Federal Aviation Administration . *Angle of Attack Awareness*. 2024.

7. Gebel, Achim. "The aviation industry." The business of sustainability: Building industry cases for corporate sustainability. London: Palgrave Macmillan UK, 2004. 99-123.

8. GlobeAir AG. “What Does “Laminar Flow” Mean? • GlobeAir.” *GlobeAir*, 2024, www.globeair.com/g/laminar-flow#:~:text=Drag%20Reduction%3A%20Laminar%20flow%20is. Accessed 11 Aug. 2024.

9. Hall, Nancy. “Lift to Drag Ratio.” *Glenn Research Center* | *NASA*, 21 July 2022, www1.grc.nasa.gov/beginners-guide-to-aeronautics/lift-to-drag-ratio/.

10. International Air Transport Association. “Our Commitment to Fly Net Zero by 2050.” *Www.iata.org*, 2024, www.iata.org/en/programs/sustainability/flynetzero/.

11. International Civil Aviation Organization Agency. *Background (until the Mid-1980s)*. 2013.

12. Kabir, Mohammad Asif, et al. *Numerical Assessment of the Backward Facing Step for NACA 0015 Airfoil Using Computational Fluid Dynamics*. 3 May 2019.

13. L., Nathan . “Lexy Throws a Dart with an Initial Velocity of 25 Ms at an Angle of 60 Degrees Relative to the Ground. What Is the Approximate Vertical Component of the Initial Velocity? | Socratic.” *Socratic.org*, 16 June 2017, socratic.org/questions/lexy-throws-a-dart-with-an-initial-velocity-of-25-ms-at-an-angle-of-60-degrees-r.

14. Maverick, J.B. “How Is Sensitivity Analysis Used?” *Investopedia*, 28 Mar. 2022, www.investopedia.com/ask/answers/052115/what-are-some-examples-ways-sensitivity-analysis-can-be-used.asp#:~:text=Sensitivity%20analysis%20is%20used%20to.

15. Overton, Jeff. “Fact Sheet: The Growth in Greenhouse Gas Emissions from Commercial Aviation | White Papers | EESI.” *Www.eesi.org*, Environmental and Energy Study Institute, 9 June 2022, www.eesi.org/papers/view/fact-sheet-the-growth-in-greenhouse-gas-emissions-from-commercial-aviation.

16. NASA. “Velocity Effects on Aerodynamic Forces.” *Www.grc.nasa.gov*, 2024, www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/vel.html.

17. Ritchie, Hannah. “What Share of Global CO_2 Emissions Come from Aviation?” *Our World in Data*, 8 Apr. 2024, ourworldindata.org/global-aviation-emissions#:~:text=Aviation%20accounts%20for%202.5%25%20of.

18. scikit-learn. “1.1. Linear Models — Scikit-Learn 0.24.0 Documentation.” *Scikit-Learn.org*, 2024, scikit-learn.org/stable/modules/linear_model.html.

19. Scott, D’Errah. “Net-Zero Aviation: How It Started and How It’s Going.” *International Council on Clean Transportation*, 15 Apr. 2024, theicct.org/net-zero-aviation-how-it-started-and-how-its-going-apr24/#:~:text=In%202022%2C%20the%20International%20Civil.

20. SIMSCALE. “What Is Lift, Drag and Pitch? | SimWiki.” *SimScale*, 1 Sept. 2023, www.simscale.com/docs/simwiki/lift-drag-pitch/.

21. skybrary. “Angle of Attack (AOA) | SKYbrary Aviation Safety.” *Skybrary.aero*, 2024, skybrary.aero/articles/angle-attack-aoa#:~:text=An%20increase%20in%20angle%20of.

22. Topham, Gwyn. “Airbus Forecasts That Number of Planes in Sky Will Double in 20 Years.” *The Guardian*, The Guardian, 18 Sept. 2019, www.theguardian.com/business/2019/sep/18/airbus-forecasts-48000-aircraft-to-be-in-operation-by-2038#:~:text=Airbus%20forecasts%20that%20number%20of%20planes%20in%20sky%20will%20double%20in%2020%20years. Accessed 10 Aug. 2024.