Simulation and Optimization of Hybrid Chemical-Electric Propulsion Transfer from Low Earth Orbit to Geosynchronous Earth Orbit

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Abstract:

Recent advances in satellite deployment and increasing demands for efficient use of propellant have prompted interest in transfer strategies from low Earth orbit (LEO) to geosynchronous Earth orbit (GEO). This paper compares hybrid chemical-electric propulsion with traditional single-mode chemical and electric systems. Using Python simulations, it models LEO-to-GEO transfers with combinations of high-thrust chemical engines and low-thrust electric thrusters, calculating values for both transfer time and propellant consumed. We hypothesized that the hybrid approach would yield a reduction in propellant mass, while keeping travel times closer to those achieved by chemical propulsion alone. Simulation results confirm this hypothesis: hybrid strategies outperform electric alone in time, and chemical alone in propellant use. These findings suggest hybrid propulsion as an adaptable solution for contemporary satellite missions, balancing efficiency with operational constraints.

LEO- GEO Transfer Background Study:

Geosynchronous Earth orbit (GEO) is utilized in surveillance, navigation, communication, and weather monitoring. The GEO has the same revolution period as Earth's rotation time and, therefore, the satellites do not appear to move from the ground. They cover large regions and collect data for applications ranging from storm tracking to global internet services.

Satellites are traditionally launched into a circular low Earth orbit (LEO) at an altitude of 160 km - 2,000 km (for this study 622km), followed by a transfer into a highly elliptical geosynchronous transfer orbit (GTO) and then circularized at GEO (35,000 km above mean sea level) using chemical, impulsive propulsion at each maneuver [1] (NASA Science). Typical transfer sequences follow the pathway: launch to LEO, execute a large burn (delta-v) to reach GTO, and then use the spacecraft's apogee motor for a final burn at GTO apoapsis to enter GEO.

Traditional transfers rely on chemical propulsion, which results in high propellant requirements and limited flexibility. Conversely, purely electric propulsion, although efficient, results in a longer transfer duration, which can impact mission timelines. Hybrid propulsion addresses these challenges by allowing mission planners to tailor trajectories in the context of a specific mission.

There are two primary types of electric thrusters employed in satellite orbit raising: ion thrusters and Hall-effect thrusters. Ion thrusters use electrostatic fields and grids to accelerate positively charged ions to extremely high velocities. This design enables low impulse, higher propellant efficiency, and lower mass consumption over prolonged time spans. However, ion engines typically generate very small thrust, making them better suited for deep-space missions and slow, gradual orbit changes [2][3][4].

On the other hand, Hall-effect thrusters utilize a radial magnetic field in conjunction with an axial electric field to trap electrons, subsequently accelerating ions through a plasma channel, which results in a higher thrust-to-power ratio.

This allows Hall thrusters to provide higher thrust per unit of electrical input compared to ion thrusters, and makes them increasingly popular for near-Earth applications (such as LEO-to-GEO transfers) where maneuver timelines and operational practicality are critical [2] [4] [5]. Studies evaluating LEO-GEO transfers consistently show Hall thrusters are advantageous, delivering high payload fractions and reasonable transfer durations with efficient propellant use [4][1], so this study assumes a Hall thruster propulsion system.

Several GEO-bound satellites have now used fully electric propulsion for orbit raising. Commercial missions commonly rely on electric propulsion to move from geosynchronous transfer orbit (GTO) to GEO, as with the landmark Eutelsat 115 West B satellite, which used four XIPS ion thrusters in place of a chemical apogee kick motor [6][7]. These orbits are achieved using extended, low-thrust, high-specific-impulse spiral trajectories to gradually raise and circularize the satellite's orbit [3][2]; however, this method means a long transfer time and results in more time spent in the Van Alan radiation belts.

Solar electric propulsion has also been demonstrated in smaller satellites and technology demos, where high-efficiency ion or Hall-effect thrusters progressively maneuver the spacecraft from elliptical orbits to their final positions [3][2]. Although direct, fully electric LEO-to-GEO transfer remains rare due to long transfer times, increased radiation risk, and conjunction risk with debris, mission concepts and agency studies have established its feasibility for the future.

<u>Theory and Equations:</u>

We model the spacecraft transfer assuming a two-body system with no Earth perturbations, gravity harmonics, or atmospheric drag. Keplerian propagator mechanics are used for impulsive burns, while low-thrust segments are treated as continuous thrust with constant specific impulse throughout the maneuver.

Delta-V from the Rocket Equation:

This equation determines the total velocity change, delta-v, achieved by a burn, forming a direct link between the spacecraft's initial and final mass, engine efficiency, and maneuver sizing. It sizes all impulsive maneuvers in the transfer and is used as the basis for all subsequent mass and burn calculations in this study.

$$\Delta v = I_{sp} * g_0 * ln(\frac{m_0}{m_f})$$

Where Δv is the total change in velocity (km/s), I_{sp} = specific impulse (s), g_0 = standard gravity (9.80665 m/s^2), m_0 = mass before burn (kg), and m_f = mass after burn (kg)

Rocket Equation for Mass:
This equation rearranges the rocket equation to solve for the spacecraft's final mass after spending a given impulsive delta-v. It is used at each stage to update the spacecraft's mass after an impulsive burn, ensuring propellant depletion is correctly tracked across all phases of the mission, and is given by:

$$m_{final} = m_{initial} * exp(-\frac{|\Delta v|}{I_{sp} * g_0})$$

where m_f = final mass after burn (kg), m_0 = initial mass before burn (kg), Δv = velocity change (km/s), I_{sp} = specific impulse (s), and g_0 = standard gravity (9.80665 m/s^2)

Continuous Mass Loss for Low-Thrust:

To model the gradual depletion of mass when thrust is applied continuously (such as during electric propulsion), this differential equation is integrated throughout the low-thrust segment.

$$\frac{dm}{dt} = -\frac{T}{I_{sp} * g_0}$$

Where, $\frac{dm}{dt}$ = instantaneous mass loss rate (kg/s), T = thrust (N), I_{sp} = specific impulse (s), and g_0 = standard gravity (9.80665 m/s^2)

Velocity in Circular LEO:
This equation is used to compute the starting velocity for the spacecraft in a circular low Earth orbit. In this study, the equation serves as the initial condition for all subsequent maneuver planning and propagation.

$$v_{LEO} = \sqrt{\mu / r_{LEO}}$$

Semi-Major Axis of GTO:
All geometric and timing calculations for transfers between LEO and GEO depend on the semi-major axis of the transfer (Geostationary Transfer Orbit) ellipse.

$$a_{GTO} = \frac{r_{LEO} + r_{GEO}}{2}$$

Transfer Time - GTO Apoapsis to Periapse:

The time for a spacecraft to traverse its transfer arc is derived from Kepler's laws. This formula is used to estimate the duration of the main coast phase:

$$t_{GTO} = \pi \sqrt{\frac{a_{GTO}^3}{\mu}}$$

Where t_{GTO} = time of flight (seconds), a_{GTO} = semi-major axis of the transfer ellipse (km), and μ = gravitational parameter (km^3/s^2)

Jacobian:
The Jacobian matrix is used for sensitivity and to speed convergence in the optimizer. The Jacobian matrix summarizes how small changes to the optimization variables (such as burn timing and velocity corrections) affect the final state constraints that must be matched at the end of the maneuver. The Jacobian matrix can be described by:

$$J = \frac{\partial F}{\partial p}$$

For this study, the matrix is specifically a 4-by-5 array: each of the four rows corresponds to a final position or velocity constraint (these are the x-position, y-position, x-velocity, and y-velocity that the spacecraft must achieve at its final orbit). Each of the five columns corresponds to an optimization parameter (the initial y-velocity at the start of the transfer, the time of flight for the low-thrust segment, the delta-v applied in the x direction at the endpoint, the delta-v applied in the y direction at the endpoint, and a fifth parameter is for an additional control input or maneuver variable, providing extra flexibility to the optimizer as needed for future expansions or more complex mission constraints).

By evaluating the partial derivatives for all combinations of these variables and constraints, the Jacobian allows the optimizer to predict and course-correct. This ensures every free variable is fine-tuned so that the assembled trajectory meets mission targets at GEO.

<u>Propagation and Optimization Methodology (Computational Methods):</u>

Python's NumPy library forms the backbone of all calculations, managing core vector arithmetic (such as orbital state vectors and tangential thrust directions) and powering the matrix operations needed for propagator construction. All velocities, positions, and masses are stored as flattened arrays, which allows direct, element-wise manipulation for optimization and plotting.

Trajectory propagation is divided by propulsion type. For impulsive (instantaneously applied) burns, such as LEO to near GTO and final GEO insertion, the code uses a Keplerian propagator, implemented in Python using two-body Newtonian mechanics. Here, the function accepts vectors for initial position and velocity and returns their evolution over time. The code's structure allows for easy switching between integration times and analyzing how the state changes when maneuver parameters are adjusted. Impulsive maneuvers, such as the initial burn to enter GTO and the final burn to circularize at GEO, are modeled with a Keplerian propagator function (written in Python), which implements closed-form solutions for elliptical orbits in a two-body problem.

The low-thrust segment, which models the continuous spiral, demands a more advanced solution. This part uses a purpose-built function, *low_thrust_propagator_2D*, that embeds SciPy's Dormand–Prince (DOP853) integrator. This approach divides the total transfer time into 1,000 evenly spaced time steps, capturing changes in mass and velocity as thrust is applied at each interval throughout the low-thrust segment. Mass flow is constantly tracked, so propellant loss is accounted for at each step, and all intermediate states (such as positions and velocities) are recovered for further analysis. Here, the simulation keeps track of the spacecraft by solving six coupled ordinary differential equations: two for position, two for velocity, one for remaining mass, and one for time. This explicit integration approach for low-thrust segments closely parallels the step numerical propagation methods used by Vavrina and Howell, who demonstrated their effectiveness for complex low-thrust mission planning [8].

To target the desired outcome, the code implements a shooting method, which iteratively refines the guess for initial control variables until the trajectory matches the required boundary conditions. At each loop, the shooting method propagates the full trajectory from initial guess to the end, computes the disparity in the final state ("residuals"), and feeds this back into the optimizer. For sensitivity, the code propagates State Transition Matrices (STM) alongside the trajectory, enabling calculation of the Jacobian.

Optimization is carried out using SciPy's minimize routine, specifically the SLSQP algorithm, because it can accept analytic Jacobians and handle nonlinear constraint equalities. Here, nonlinear constraints link the launch parameters, maneuver times, and velocity corrections to the final required position and velocity at GEO. The objective is to minimize total propellant burned, using callback functions to log convergence progress. Each optimization run solves for the free variables that relate the spacecraft's actual arrival state with the theoretical target at GEO, with the shooting method adjusting those input guesses until the position, velocity, and mass all match within defined tolerances. Sensitivity is informed by propagating state transition matrices, which compute the full Jacobian at the trajectory endpoint.

This approach draws on recent advancements in hybrid trajectory optimization as developed by Taheri et al. [9], where the overall transfer is divided into sequential segments corresponding to different propulsion modes. Like their methodology, this study employs a stepwise framework: the trajectory is broken into impulsive (chemical) and continuous (electric) phases, with each segment's boundary conditions and control variables defined independently and the optimizer iteratively refining all free parameters so that the assembled phases yield a continuous, feasible path that minimizes total propellant consumption while meeting arrival constraints at GEO.

Throughout, all simulation parameters - like gravitational constant, initial orbit radii, specific impulse, thrust - are passed through argument lists, so each run can be quickly adapted for alternative spacecraft or target orbits. The thrust parameter used in this study reflects the SETS ST-40 Hall thruster, which was designed for the propulsion of LEO–GEO satellites and offers multi-propellant capability and flexible thrust and specific impulse, with published ISP values typically ranging from 1,500 to 2,000 s (1,500s for this study) and 250–600 mN (200mN in this study) thrust [10].

The final results are visualized using Matplotlib, where 2D trajectory plots, mission mass loss profiles, transfer time comparisons, and optimization progress are built using Python's plotting calls.

Results

The results of the hybrid trajectory optimization procedure demonstrate the effectiveness of combining high-thrust and low-thrust propulsion for transfers from low Earth orbit (LEO) to geostationary orbit (GEO). The transfer was carried out from an initial LEO at a radius of 7000 km to a target GEO radius of 42,164 km, starting with a spacecraft mass of 8000 kg. Using a sequence of two high-thrust (impulsive) maneuvers with an intermediate low-thrust arc, the code computed the required burns and overall mass consumption.

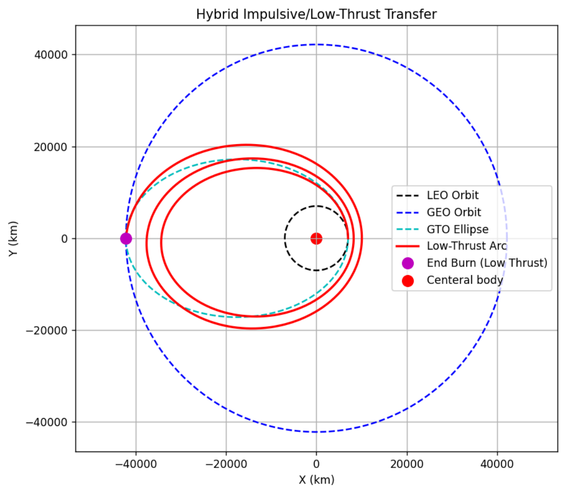


Figure 1: *Transfer trajectory plot of low-thrust segment from Near GTO to GTO*

The first impulsive maneuver at LEO resulted in a significant mass loss, reducing the mass from 8000 kg to approximately 1288 kg: a decrease of 6712 kg due to the high propellant requirement of the initial burn. Following this, the spacecraft performed a low-thrust transfer, which was notably efficient in terms of propellant use. The mass loss during the low-thrust segment was only about 0.058 kg.

A final impulsive burn at GEO completed the transfer, using an additional 778.6 kg of mass. Summing all stages, the total propellant consumed throughout the mission was approximately 7490.6 kg. Trajectory and mass profiles generated by the simulation further illustrate these sharp drops in mass at the impulsive burns and the gradual mass decrease during the low-thrust arc.

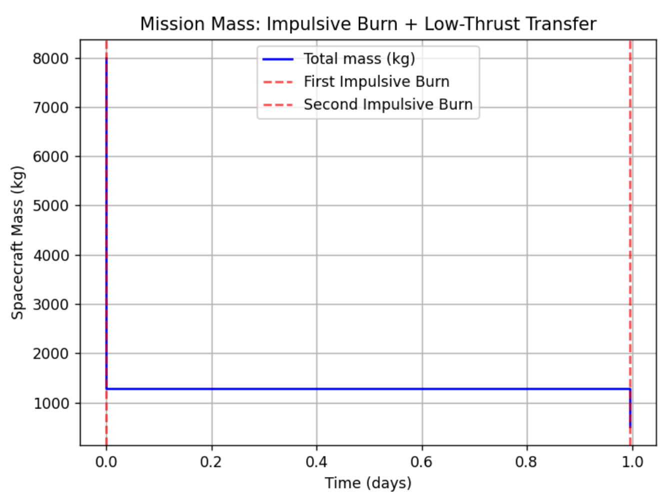


Figure 2: *Satellite Mass Profile over days (not including coasting time)*

The optimizer converged after 21 iterations, successfully minimizing the mass while meeting trajectory and velocity constraints.

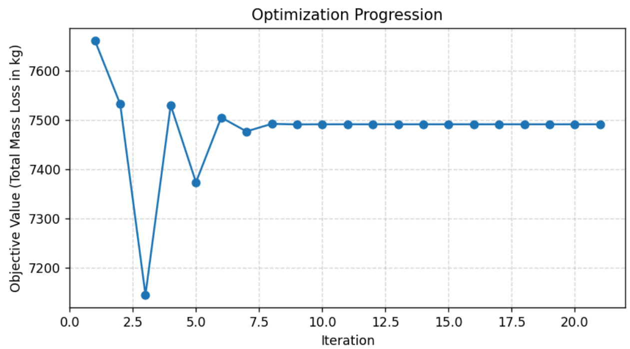


Figure 3: *Optimization progress Δm with SciPy SLSQP method*

Discussion

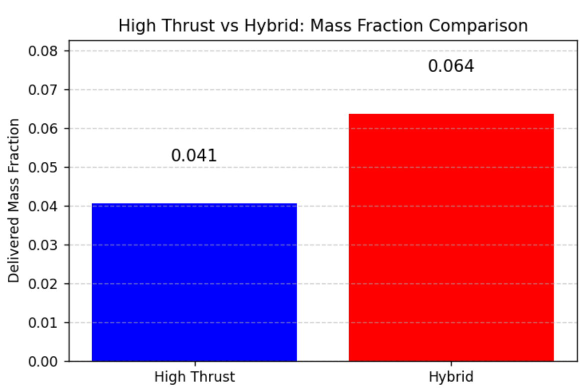
The initial high-thrust burn contributed the largest share to overall propellant expenditure, consuming approximately 6712 kg out of the initial 8000 kg spacecraft mass. Upon transitioning to the low-thrust phase, propellant consumption decreased dramatically, with mass loss limited to just 0.058 kg.

The high-thrust burns are responsible for the majority of the mission's mass usage, while the low-thrust segment demonstrates propellant savings. This is in contrast to strategies limited to exclusively high- or low-thrust, which typically offer less favorable trade-offs between fuel usage and mission length.

To visualize the mass loss difference between a purely high thrust mission versus the hybrid mission, the mass fraction of both was calculated using the equation:

$$m_{fraction} = \frac{m_f}{m_0}$$

Where m_f is the final mass and m_0 is the initial mass (8000 kg)



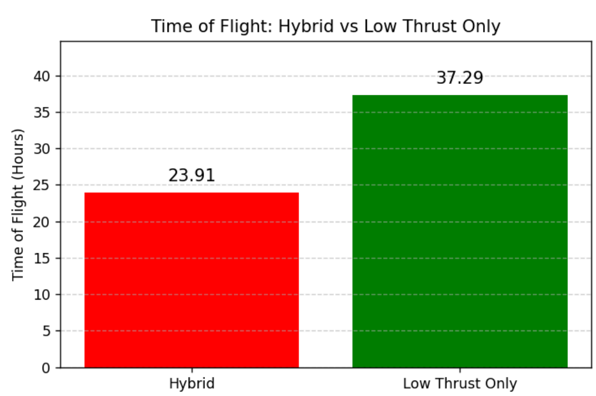


Figure 4: *Mass Fraction Comparison* **Figure 5:** *Transfer Time of Flight Comparison*

While the change in delivered mass fraction may appear modest (rising from 0.041 for a pure high-thrust maneuver to 0.064 for the hybrid strategy), this improvement is meaningful in the context of spacecraft design. The mass fraction increase represents a 56% relative gain in the amount of payload that can be delivered to the target orbit, which can be crucial for missions where every kilogram carries high value. For example, a satellite with an initial mass of 8000 kg using a hybrid transfer could deliver approximately 184 kg more to its final orbit compared to a purely high-thrust trajectory.
Additionally, Figure 5's comparison of time of flight reveals that hybrid transfers offer shorter operational timelines than low-thrust-only maneuvers. The hybrid trajectory requires approximately 23.9 hours to complete the orbital transfer, whereas the low-thrust-only mission requires 55.91% more time at 37.3 hours. This difference demonstrates that incorporating even a single high-thrust maneuver can greatly shorten mission timelines.

However, this work is subject to several idealization limitations. The models have perfectly uninterrupted propulsion and do not account for perturbing influences, such as non-Keplerian gravitational effects or spacecraft attitude constraints, which are significant concerns for operational mission planning. Addressing these factors, along with extending the approach to multi-revolution transfers or scenarios involving more complex boundary conditions, will result in more realistic results. Even so, the findings presented here emphasise the practical advantages of hybrid propulsion architectures in contemporary trajectory design.

Conclusions

This study demonstrates the effectiveness of hybrid propulsion strategies for transferring spacecraft from low Earth orbit (LEO) to geosynchronous Earth orbit (GEO). By simulating an impulsive-initiated, low-thrust-spiral trajectory within a Python-based optimization code, the approach balances the strengths of chemical and electric propulsion, demonstrating that a low-thrust transfer is capable of delivering a spacecraft from LEO to GEO without reliance on high-thrust, impulsive maneuvers.

The simulation confirmed that hybrid approaches deliver a meaningful increase in delivered payload fraction compared to purely high-thrust transfers, with a 56% relative gain observed in this study, and a 36% decrease in relative time of flight compared to a purely low-thrust trajectory. Mission planners can tune the split between chemical and electric burns to meet a range of constraints, optimizing for fastest arrival, lowest mass, or intermediate mission goals.

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