Application Of Modified Newtonian Mechanism for Oort Cloud objects

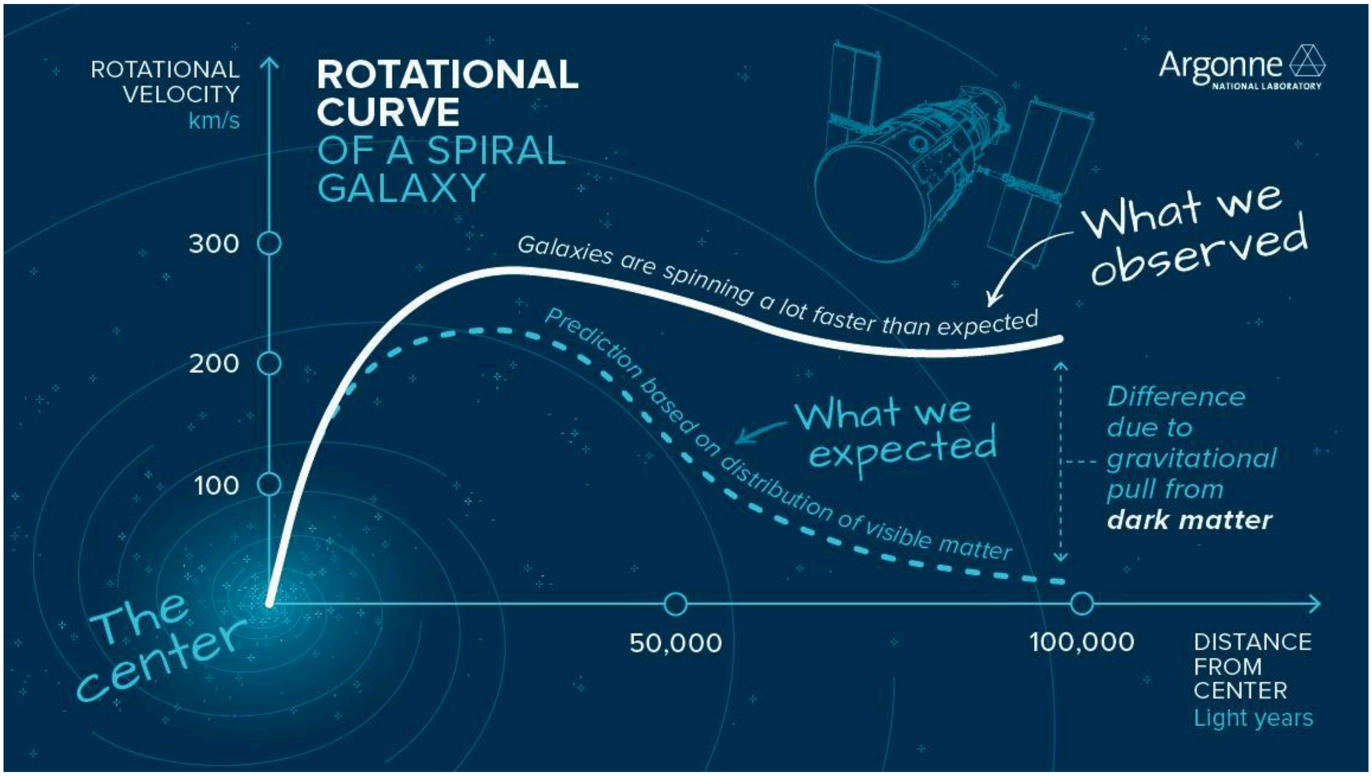
Yusong Wang && Zilin Zhang (in alphabetical order)

Abstract

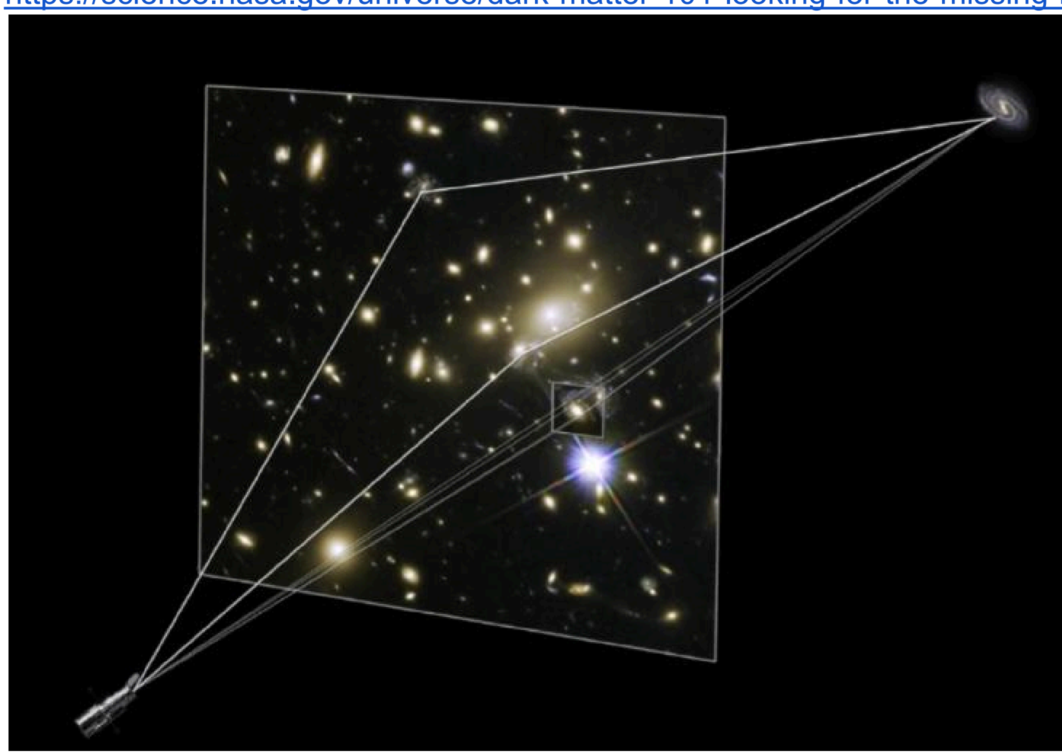
This paper investigates the applicability of modified Newtonian mechanics(MOND) to celestial body inside solar system, specifically by studying the trajectory of the Oort Cloud comets with aphelion distance more than10^3 AU.Although Modified Newtonian Mechanics been widely proposed as an alternative explanation to dark matter for the missing mass problem in the large galaxy. Notably in the spiral galaxy where the galaxy mass shows significant discrepancies with observation with respect to the magnitude velocity of the star at the edge of the galaxy. Using computer simulation we model the orbit motion of comet under both MOND and classical newtonian conditions .Through analyzing ,we identified measurable discrepancies in velocity component for the comet .The results suggest that although the MOND itself has negligible impact for inner solar system object ,it processed significant impact on comet orbit when the aphelion distance were more than 10^3 AU ,and became increasingly large for longer aphelion distance .Our finding provide a insight of MOND potential impact of observable solar system impact that may could possibly testing this derivation under future observation.

Introduction

The model of "Modified Newtonian dynamics" abbreviated as MOND ,first proposed by Milgrod was a hypothesis that developed to address the discrepancy between the observable mass and the galactic rotational curve under the prediction of modified Newtonian mechanics (Trippe, S. (2014).https://doi.org/10.48550/arXiv.1401.5904).Specifically,MOND suggested that gravity functioned differently under extreme low acceleration,so it introduced a modification that increased the gravitational constant under extreme low acceleration . It served as an alternative solution to this problem beside theoretical Dark matter .Which suggested there existed new particles which only reacted to gravity that increases the gravitational mass and thus give a reasonable answer to the missing mass problem .((Oks, E. (2021). https://doi.org/10.1016/j.newar.2021.101632)Although lack experiment confirmation ,the theories of black matter was supported by observations evidence such as gravity lensing .The gravity lensing due to dark matter has been shown in the second image as light has been distorted by the relativistic effect of dark matter resulted in discrepancy of light to mass ratio for cosmological structures .



Missing mass problem :(NASA. (2018). Retrieved from https://science.nasa.gov/universe/dark-matter-101-looking-for-the-missing-mass/)



(National Aeronautics and Space Administration. (2025). https://science.nasa.gov/mission/hubble/science/science-highlights/shining-a-light-on-dark-matter)

But still ,the lack of experimental evidence of dark matter even until today s,This questioning the existence of dark matter ,that leads us into further research for the validity of both claim.

Similar to general theory of relativity that modified the mechanics under extreme high velocity ,MOND modified gravity under extreme low acceleration to fit the observations data.But the theory remain controversy .One of the major problems is the discontinuity between MOND and classical newtonian region MOND ,this could resulted in the modification of inertia ,which is not suggested through current experimental statistics.(Milgrom, M. (2025).https://doi.org/10.1103/PhysRevD.111.10403).The second problem is the empirical parameter a0 ,which is being treated as a universal constant across all scales and forms of galaxies is highly debatable.The primary reason for the concern is the its dependence on curve-fitting observation data without evidence from experiment data leads to critics about its applicability across different astronomical scenarios (Milgrom, M. (2015). https://doi.org/10.1139/cjp-2014-0211)

A foundational requirement of a physics theory is its consistency across scale ,while black hole was an exception due to the breakdown of space time at the singularity (Curiel, E. (2009–). https://plato.stanford.edu/entries/spacetime-singularities)its more due to the current limitation of observation technology . The perihelion regression for mercury orbit for an example of general relativity consistency across different astronomical scales . When consider MOND applicability across different scales ,it should not only able to be applied to Galaxies orbit same its impact should be applied to solar systems bodies .So through this experiment MOND should be not only be valid for solar system object it will also should be consistent in MOND dominated region and newtonian dominated region .Under this general idea , our main focus is the object that transit between MOND and Newtonian region to simulate the MOND impact on its trajectory to test its consistency.

Our main subject of simulation is Oort Cloud ,extending from 2x10^3 AU to 10^4 AU from the sun which constitutes a region where the MOND effect becomes nonnegligible. In this region existed a significant amount of objects that could be under the influence of MOND theorem (National Aeronautics and Space Administration. (2025).https://science.nasa.gov/solar-system/oort-cloud/) . Although currently it's impossible for direct observation due to the dimness and lack of thermal signal and reference object ,super long period comet originate from the Oort Cloud provide a method to indirectly study their dynamics across different region.Through data of existing comet and computer simulation we can simulated its dynamics of the object that cross over both MOND and classical newtonian mechanics region making them ideal candidate for investigating the behavior of object through different region.

Although one notable discrepancy is the gravitational influence from stars outside the solar system which its gravitational force being amplified by the MOND that may result in an orbit longer than Kepler's law suggested .Although we will still analyze data from comet with longer orbit this will limit the range of our simulation to not exceed 3000 AU so we could neglect this discrepancy .But more importantly when we consider a comet orbit from inner solar system to edge of Oort cloud .Using comet west for example of the demonstration which has a semi major axis about 6300 AU.When calculated using Kepler's third law by using its estimate orbit period of 500000 years (Oxford University Press. (2011). https://www.oxfordreference.com/display/10.1093/oi/authority.20110803121806900) from Oxford research .We derived its semi major axis using the formula shown below .

$$T^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

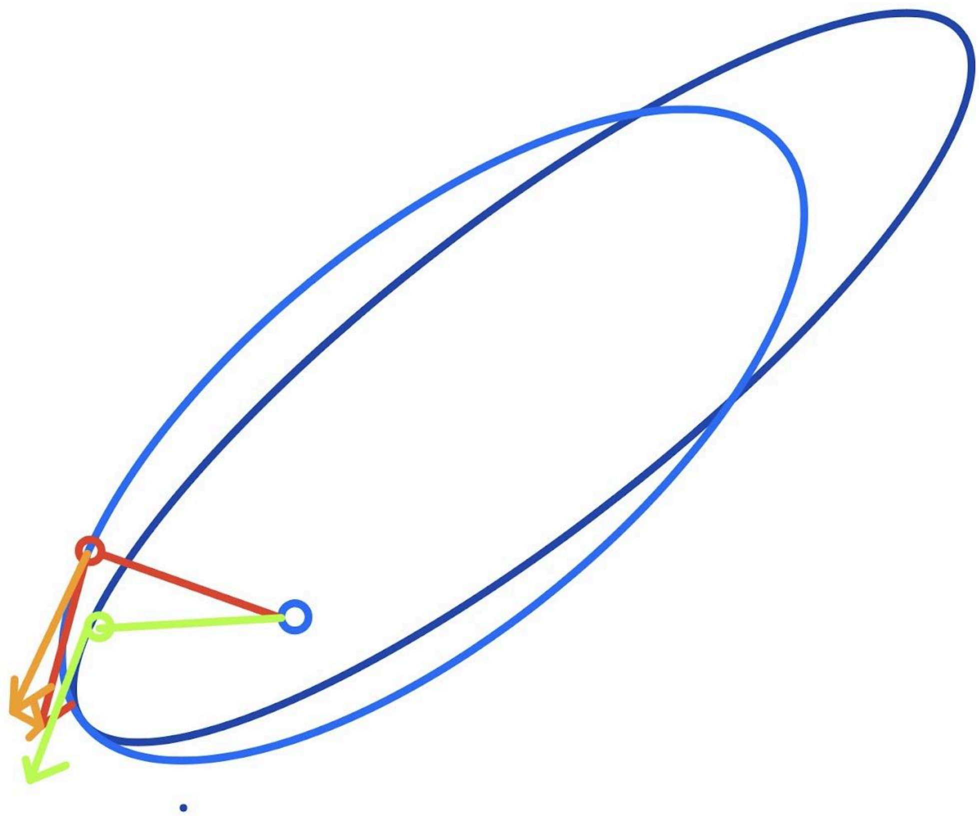
$$T^2 = a^3$$

T= Earth years
a= astronomical units (1a= one Earth year)
M= solar mass
G= universal gravitational constant

(Gordon, A. (2012). https://renpkepler.weebly.com/the-law-of-periods.html)

Through our calculations,the gravitational acceleration at the outer edge of orbit was closed with the MOND acceleration constant(1.2x10^-10 m/s^2). This would resulted in higher magnitude of gravitational force from sun than previously expected .When consider its impact on the solar system object trajectory, Kepler's law states that the potential energy and angular momentum is conserved .This shown that although angular momentum is still conserved due to the zero net torque provided by gravity, increasing gravitational force will leads to increasing in magnitude of velocity .This eventually increase its non-tangential component of velocity due to the conservation of tangential part.Under this scenario MOND modified orbital shape and eccentricity which will be considered in our following paragraph .

Under MOND the unmatched speed will happens in outer section ,it resulted in outer orbit will be bring closer to the sun resulted in a" flattened" trajectory compared to the classical newtonian mechanics as we explain above .The discrepancy between MOND and classical newtonian mechanics has been shown in the graph below .Respect to the observer instead of the sun The x component of the velocity will decrease and the y component will increase as energy must be conserved .(the graph is not drawn to scale it's only a demonstration without loss of generality)



Dark blue:classical newtonian orbit
Light blue:MOND orbit

The trajectory will therefore not exhibit traditional elliptical orbit of celestial bodies rather than be elongated and exhibit a teardrop-shaped trajectory that resulted in different dynamics pattern than we perilously expected .Which will show discrepancy in dynamic across different regions from the classical Newtonian model predict .

In this study ,using computer simulation we examined the different potential orbit of Oort Cloud Object with different semi major axis .Through comparing different velocity component and different heliocentric distance ,we would be able to verify the different orbit characteristics of both senecio .This could examined that whether MOND could produce measurable deviation in velocity component that provide base for evaluation of the applicability of MOND when its came beyond galactic scale.
Here is the list of notable object that’s believed to be originate from the Oort Cloud:

(Astronomy. (n.d.).Comet West reached peak brightness in March 1976. https://www.astronomy.com/today-in-the-history-of-astronomy/feb-25-1976-comet-west-reaches-perihelion/)

Comet West (C1975 V1) was discovered in 1975/8/10, by Richard M. West. The perihelion distance is approximately 0.197AU, while the estimated aphelion distance exceeds 7 x 10^3 AU. The orbital eccentricity is 0.99997. The orbital period is estimated to be approximately 558,000 years . The estimated nucleus diameter is approximately 5-10km.The maximum brightness of the comet is about magnitude was -3. This is a long-period comet that entered the inner solar system from the Oort Cloud .

(Astronomy. (n.d.). Remembering Comet Hale-Bopp.
https://www.astronomy.com/observing/remembering-comet-hale-bopp/)

Comet Hale-Boop (C/1995 O1) was discovered in 1995/7/23, by Alan Hale and Thomas Boop. The perihelion distance is 0.91741 AU , while the aphelion distance is 363.18653AU. The orbital eccentricity is 0.99496. The orbital period is estimated to be approximately 2533 years. The estimated nucleus diameter is approximately 60km. The comet's magnitude was -1.4 at perihelion. During its perihelion passage in 1997, the comet displayed both a dust tail and an ion tail. Its highly eccentric orbit suggests that it is a long period comet originating from the outer Oort Cloud.

Comet Hyakutake (C/1996 B2) was discovered in 1996/1/31, by Japanese amateur astronomer Yuji Hyakutake. The perihelion distance is 0.2301987 AU, while the aphelion distance is 3.41 x 10^3 AU . The orbital eccentricity is 0.9998946. The orbital period is estimated to be approximately 70,000 years. The estimated nucleus diameter is approximately 4.2km. In 1996 the comet passed relatively close to earth at a distance within 0.1AU. It came from the Oort Cloud.

Comet McNaught (C/2006 P1), a Non-periodic comet, discovered in 2006/8/7, by Robert H. McNaught. The perihelion distance is 0.1707 AU , the aphelion distance is not an accurate estimate. The orbital eccentricity is 0.99917. The orbital period is estimated to be approximately 92600 years. The estimated nucleus diameter is approximately below 25km. During its perihelion passage in 2007, the comet reached a maximum brightness of approximately magnitude -5. It displayed a large fan-shaped dust tail extending up to 27.36AU

Methods

The goal of this experiment is to discover the range of applicability of MOND to extend beyond Galaxies orbit into the celestial bodies inside solar system.Our main subject is Oort Cloud comet trajectories r>1*10^3 AU. By recording the orbital data of the simulated Oort Cloud object ,the study aims to determine whether MOND has any potential impact on celestial bodies in the solar system.

Device being used
Universal sandbox version 6.1
(No mode is added)
Device:Apple Mac book air
Version:macOS Tahoe 26.3.1
Microchip:Apple M2
Storage :Macintosh HD
Memory :GB

Experimental constant :
Independent variable :gravitational constant instead of a constant its would varied with distant
Dependent variable:Velocity of the comet at a certain distance from the sun.
Constant:

1. Mass of sun
2. Perihelion distant of the comet
3. Structure and velocity of the comet

Experiment procedure :

1. The MOND model we are using was the original paper published by Milgorod
2. We only use sun as sole source of gravity in our simulation
3. Using Kepler's law that a comet orbit was being uniquely determined by using comet perihelion velocity and distance therefore through setting up comet perihelion distance and velocity ,we could determine its orbit under classical Newtonian mechanics and Modified Newtonian mechanics .
4. For the classical Newtonian model the gravitational constant will be held unchanged ,At 50/100/200AU ,the velocity component of the comet after one orbit will be measured .
5. The experiment will be conducted 5 times, and results will be presented in a table .
6. For Modified Newtonian Mechanics We would divide the orbit trajectory with radial distance more than 700 AU from the sun evenly into 9 segments By the comet’s velocity at different distances along its orbit, we use the distant 700AU from the Sun since at 700AU the discrepancy’s of gravitational acceleration between MOND and classical

newtonian mechanics will reach a significant number ~1%, thereby influencing the comet trajectory.

7. We will deleted all planet to exclude the disturbance in planet orbit caused by increasing gravitational constant since this may leads to potential disruption in stability of solar system
8. Each experiment will be repeated for 5 times ,and for each individual experiment the starting point will be added 20 AU to the distance for example first time 7 x 10^2 AU second time 7.2 x 10^2 AU etc .
9. The velocity component of the comet from 50/100/200 AU in each experiment will be collected from the sun manually and averaged to reduce discrepancies of unsmooth curve due to limitations of the experiment device.
10. For Modified Newtonian model.The whole experiment will be repeated for 5 times to reduce human error ,system error.
11. The perihelion velocity and distance and velocity at 5 AU from the sun after one orbit and being collected into a spread sheet .

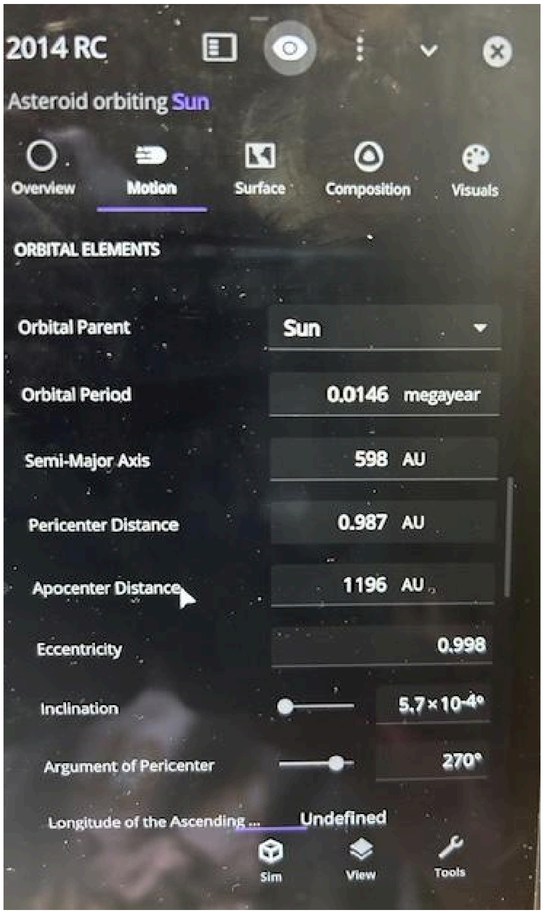
Limitation:

1. Due to the limitation of the simulation app and performance of the computer ,we are incapable of presenting a smooth change in the gravitational constant.
2. eight planet is excluded ,although it’s unlikely for any of them to produce a significant impact since planet were able to clean its neighborhood meaning if a comet across any planet its would be crashed or ejected that its orbit disturbed ,due to the limitations of device we are unable to consider them into model
3. The simulation is non-relativistic meaning it didn’t consider the influence of curvature of space resulting in discrepancies when the comet is close to the sun.
4. The potential impact of other Oort Cloud Objects and potential influence from dwarf planets was not being concluded .
5. The MOND was continuously developing include relativistic model like TeVeS which will not be considered in the experiment
6. The influence from asteroid belt was not considered
7. The simulation app itself was limited and couldn’t fully simulated the structural change in comet when its closed to the sun
8. There are numerous other celestial bodies that existed in the Kuiper belt ,asteroid belt and Oort Cloud .If any of this object makes a close pass it may result with unexpected gravitational disturbances although very unlikely considering its vastness and dispersed.

Result

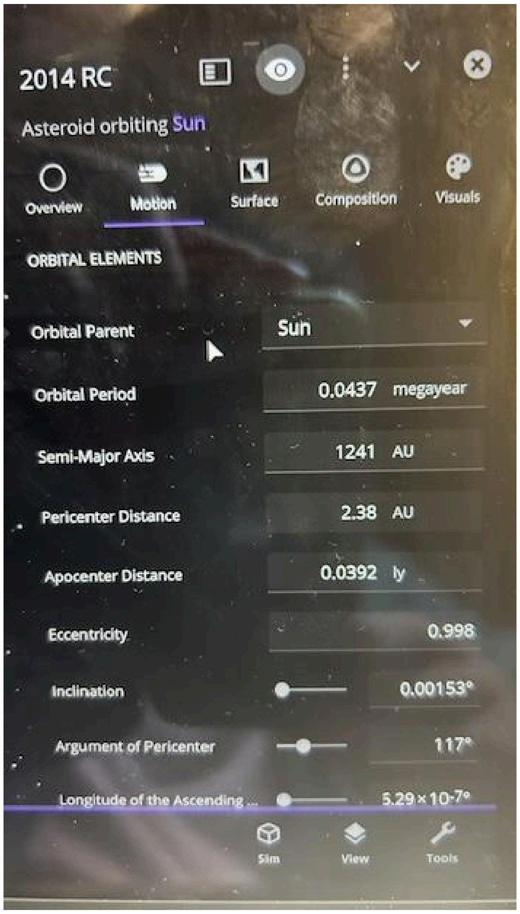
For Newtonian object

The setting for the approximate 1.2 x 10^3 AU aphelion distant comet



The data recorded at 50/100/200AU data point at each of the five experiment

	50AU	100AU	200AU
1	2.07km/s/5.46km/s	1.60km/s/3.71km/s	1.16km/s/2.45km/s
2	2.07km/s/5.45km/s	1.60km/s/3.70km/s	1.17km/s/2.46km/s
3	2.07km/s/5.45km/s	1.60km/s/3.70km/s	1.16km/s/2.46km/s
4	2.07km/s/5.46km/s	1.61km/s/3.71km/s	1.16km/s/2.46km/s
5	2.07km/s/5.45km/s	1.60km/s/3.71km/s	1.17km/s/2.45km/s



The setting of approximately aphelion distant of 2.5 x 10^3 AU object with setting above

	50AU	100AU	200AU
1	3.73km/s/4.58km/s	2.39km/s/3.38km/s	1.53km/s/2.41km/s
2	3.73km/s/4.58km/s	2.39km/s/3.38km/s	1.53km/s/2.41km/s
3	3.73km/s/4.58km/s	2.39km/s/3.37km/s	1.52km/s/2.42km/s
4	3.73km/s/4.59km/s	2.39km/s/3.38km/s	1.52km/s/2.41km/s
5	3.73km/s/4.58km/s	2.39km/s/3.38km/s	1.53km/s/2.43km/s

Data record at each data point for comet under the conditions of modified Newtonian mechanics
:

1.Velocity component data at each data point for 1200AU comet under MOND scenario :

	50AU	100 AU	200AU
700/800/900/1000/1100AU	2.07km/s/5.45km/s	1.60km/s/3.71km/s	1.17km/s/2.46km/s
720/820/920/1020/1120AU	2.08km/s/5.45km/s	1.61km/s/3.69km/s	1.17km/s/2.45km/s
740/840/940/1040/1140AU	2.08km/s/5.45km/s	1.61km/s/3.69km/s	1.17km/s/2.46km/s
760/860/960/1060/1160AU	2.08km/s/5.45km/s	1.61km/s/3.70km/s	1.17km/s/2.45km/s
780/880/980/1080/1180	2.07km/s/5.46km/s	1.60km/s/3.71km/s	1.17km/s/2.46km/s

2.Velocity component data at each data point for 2500AU comet under MOND scenario

	50AU	100AU	200AU
700/1060/1420/1780/2140AU	3.69km/s/4.60km/s	2.36km/s/3.38km/s	1.51km/s/2.42km/s
720/1080/1440/1800/2160AU	3.69km/s/4.60km/s	2.35km/s/3.39km/s	1.52km/s/2.42km/s
740/1100/1460/1820/2180AU	3.68km/s/4.61km/s	2.35km/s/3.39km/s	1.51km/s/2.42km/s
760/1120/1480/1840/2200AU	3.68km/s/4.62km/s	2.35km/s/3.38km/s	1.52km/s/2.41km/s
780/1140/1500/1860/2220AU	3.67km/s/4.62km/s	2.34km/s/3.39km/s	1.52km/s/2.41km/s

Discrepancy between MOND and classical Newtonian mechanics of each scenario ,trial and data point.

	50AU	100AU	200AU
700/800/900/1000/1100AU	0%/-1%	0%/0%	+1%/+1%
720/820/920/1020/1120AU	+1%/0%	+1%/-1%	0%/-1%
740/840/940/1040/1140AU	+1%/0%	+1%/-1%	+1%/0%
760/860/960/1060/1160AU	+1%/-1%	0%/-1%	+1%/-1%
780/880/980/1080/1180	0%/+1%	0%/0%	0%/+1%

	50AU	100AU	200AU
700/1060/1420/1780/2140AU	-4%/+2%	-3%/0%	-2%/+1%
720/1080/1440/1800/2160AU	-4%/+2%	-4%/+1%	-1%/+1%
740/1100/1460/1820/2180AU	-5%/+3%	-4%/+2%	-1%/0%
760/1120/1480/1840/2200AU	-5%/+4%	-4%/0%	0%/0%
780/1140/1500/1860/2220AU	-6%/+4%	-4%/+1%	-1%/-2%

Although having different gravitational constant, under MOND conditions the sun's gravitational acceleration at 100,50,200 AU don't yield significant discrepancies compared to Classical Newtonian mechanics. But ,the X and Y component of the comet indicated that Due to the effects of MOND on its orbit trajectory ,even though the magnitude of velocity remain constant the X component will be enlarged and Y component will be reduced.This manifest a different orbital trajectory which align with our prediction in the introduction .After examined the experiment data ,we could conclude that under ideal experiment conditions MOND has a substantial impact on the comet trajectory resulted in its velocity component .

The expert data shown a 1% discrepancy for velocity component for comet with 1200AU aphelion distant respect to different heliocentric distance when compared to classical Newtonian mechanics .Also under MOND conditions,data shown 2-6% discrepancy for velocity components for 2500AU aphelion distance when compared to Classical Newtonian. This discrepancy was significantly enough to be measurable by precise equipment that could be potentially observed and verified by future study .When comparing the data from 500/100/200 AU datapoint of same trial for same object , the velocity component demonstrate greater discrepancy at closer distant ,showing the trajectory was being extorted towards its perihelion position when approaching the sun.

It’s notably that when analyzing the orbit ,even though we use the same distance to the sun,this doesn't mean they correspond to the same orbit angle,but using heliocentric distance generally creates a uniform standard for the research .So MOND actual effects on the orbit at the same orbit angle could not be significant as our simulation suggested.Its also worth mention that When comparing the data from comet with aphelion distance of 2500AU and 1200 AU ,the velocity component of 2500AU shows greater discrepancy, indicating that increasing the semi-major axis will likely contribute to higher discrepancies between MOND and classical Newtonian mechanics .

Conclusion

Under simulation conditions , the magnitude of Sun’s gravitational force at distances of 50, 100, and 200 AU is significantly greater than the empirical parameter a0 of MOND, so under this conditions the gravitational constant and the magnitude of velocity of both MOND and Classical Newtonian mechanics shown no significant discrepancy . Secondly, a comet’s orbit is uniquely determined by its perihelion distance and velocity as demonstrated by our experiment setup ,so both MOND and classical Newtonian mechanics yield identical essential orbital properties despite different gravitational constants.

Despite the condition explained above , analysis of the velocity components reveals measurable discrepancies of the orbit trajectory under MOND condition compared to classical Newtonian mechanisms . The discrepancies of orbit trajectory could be detected by future study .

Notably , for a given object, the discrepancy in velocity components is observed to increase at smaller heliocentric distances along the orbit. This indicates that the overall orbital trajectory becomes increasingly distorted toward the semi-major axis closer to the Sun. But when considering the extent of MOND effects on the orbit trajectory ,comparisons based on equal radial distances do not necessarily correspond to identical orbital angles . Using distance as a reference provides a consistent framework for analysis, but may underestimate the magnitude of variations when evaluated at equivalent orbital angles.

Furthermore, comets with aphelion distances of 1200 AU and 2500 AU indicates that larger semi-major axes correspond to greater deviations from classical predictions, implying that the magnitude of MOND effects increases with orbital scale, particularly in regions where gravitational acceleration approaches the MOND threshold.

Future direction:

Our main future target is the evolving more complex and accurate MOND model which incorporates realistic effects ,TeVeS for example.Also through more professional simulation app ,like NASA GMAT to accomplish smooth transition in gravitational constant .furthermore more powerful computer we will be able to calculate the effected of eight major planet that eventually could use along with accurate satellites observations data from research institutions that could determine MOND applicability inside solar system.

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Appendix

MOND essential equation

MOND as modified gravity

The other way to cancel the extra factor of distance in the denominator of the gravitational force law is to assume that the gravitational acceleration is the square root of the Newtonian expectation, $g \propto \sqrt{g_{\rm N}}$, with the dimensionful constant of proportionality provided by the acceleration scale a_0 so that $g = \sqrt{a_0 g_{\rm N}}$. This can only be the case at low accelerations ($g \ll a_0$), since Newtonian gravity works well at high ones. In this regime, known as the deep-MOND regime, equating the centripetal acceleration with g means that the circular speed is automatically constant (flat rotation curve)

$$\frac{V^2}{r} = \sqrt{a_0 g_{\rm N}} = \sqrt{\frac{GMa_0}{r^2}} \Rightarrow V^2 = \sqrt{GMa_0} = V_\infty^2. \tag{1}$$

MOND as modified inertia

The third way to accomplish the same is to assume that, for low accelerations, the inertial term ma changes to ma^2/a_0 for circular orbits, but the form of the gravitational force stays Newtonian. Then

$$m\frac{a^2}{a_0} = m\frac{V^4}{r^2}\frac{1}{a_0} = \frac{GMm}{r^2} \Rightarrow V^4 = GMa_0 = V_\infty^4. \tag{2}$$

leading to the same conclusion. This is MOND as modified inertia. It is apparent that the above equation of motion for a test mass on a circular orbit is indeed scale-invariant under transformations $(t, r) \rightarrow \lambda(t, r)$, and the scaling of velocity with mass only involves the product Ga_0. This is also the case for Eq. (1).

1 The A-quadratic Lagrangian (AQUAL), with the peculiar power $3/2$

A non-relativistic field equation with MOND behaviour can readily be constructed (Bekenstein and Milgrom, 1984). Since the Newtonian acceleration is the gradient of the Newtonian potential, $\vec{a}_N = -\nabla \Phi_N$, and relates to the density through the Poisson equation

$$\nabla^2 \Phi_N = 4\pi G\rho, \quad (5)$$

it implies that $4\pi G\rho = -\nabla\cdot \vec{a}_N$. Inspired by Eq. (3), one can now write

$$-\nabla\cdot\left(\mu(|\vec{a}|/a_0)\ \vec{a}\right) = 4\pi G\rho. \quad (6)$$

Demanding that the acceleration should come from the gradient of a scalar, the MOND gravitational potential Φ, such that $\vec{a} = -\nabla\Phi$, the MOND non-relativistic equation becomes

$$\nabla\cdot\left(\mu(|\nabla\Phi|/a_0)\ \nabla\Phi\right) = 4\pi G\rho \quad (7)$$

which, in the deep-MOND limit, gives

$$\nabla\cdot(|\nabla\Phi|\ \nabla\Phi) = 4\pi G a_0 \rho. \quad (8)$$

Like for solutions of the Poisson equation, the non-relativistic MOND potential is only defined up to a constant, the equation being invariant under $\Phi \to \Phi + c$ (shift symmetry). As discussed by Blanchet (2007), Eq. (7) is analogous to Gauss' law describing the electric field inside a dielectric medium, $\nabla\cdot(\mu_e \vec{E}) = 4\pi\rho_e$, where $\vec{E}$ is the electric field, ρ_e the free charge density, and $\mu_e = 1 + \chi_e$ the dielectric coefficient, where $\chi_e(E)$ is the electric susceptibility that can be a function of the amplitude of the electric field in a non-linear medium. Recasting the interpolating function as $\mu(a/a_0) = 1 + \chi(a/a_0)$, one can rewrite Eq. (7) as

$$\nabla^2\Phi = 4\pi G(\rho - \nabla\cdot\ \vec{\Pi}),$$

where $\vec{\Pi} = -(\chi \vec{a})/(4\pi G)$ is analogous to a 'polarization field'1. In particular, the susceptibility is negative, $\chi < 0$, and so there is anti-screening, the original gravitational field being enhanced.

Eq. (7) is actually also equivalent to writing

$$\mu(|\vec{a}|/a_0)\,\vec{a} = \vec{a}_N + \nabla\times\vec{A}, \tag{10}$$

and is thus equivalent to Eq. (3) in vectorial form, up to a curl field $\vec{d} = \nabla\times\vec{A}$, while it is precisely equal to it in spherical symmetry where the curl field vanishes. It is not surprising that a potential Φ, whose gradient gives the acceleration $\vec{a} = -\nabla\Phi$ that satisfies the relation $\mu(|\vec{a}|/a_0)\,\vec{a} = \vec{a}_N$, cannot in general be found, but that a correction appears. Such an acceleration would indeed also have to satisfy the inverse relation $\vec{a} = \nu(|\vec{a}_{\mathrm{N}}|/a_0)\,\vec{a}_{\mathrm{N}}$, and if $\vec{a} = -\nabla\Phi$ comes from the gradient of a potential that means that its curl is zero and so the curl of the RHS should also be zero, which implies that $\nabla|\nabla\Phi_{\mathrm{N}}| \times \nabla\Phi_{\mathrm{N}} = 0$, which is a statement about the Newtonian configuration and is not true for general configurations.

This formulation, first proposed by Bekenstein and Milgrom (1984), is the epitome of a classical modified gravity (see Sect. 1.1.2) formulation of MOND. It can be derived from an action principle. For a set of massive particles with masses m_i, with density $\rho(\vec{x},t) = \sum_i m_i \delta(\vec{x} - \vec{x}_i(t))$, where δ is the Dirac delta, and velocity field $\vec{\mathrm{v}}(\vec{x},t)$, the Newtonian Lagrangian density in the non-relativistic limit reads

$$\mathcal{L} = -\rho\left(\Phi - \frac{1}{2}\vec{\mathrm{v}}\cdot\vec{\mathrm{v}}\right) - \frac{1}{8\pi G}\mathcal{L}_{\mathrm{NG}}, \tag{11}$$

where

$$\mathcal{L}_{\mathrm{NG}} = \nabla\Phi\cdot\nabla\Phi \equiv (\nabla\Phi)^2, \tag{12}$$

and the Euler-Lagrange equations for the gravitational field Φ (leading to the Poisson equation) read

$$\partial_i\left(\frac{\partial\mathcal{L}}{\partial(\partial_i\Phi)}\right) = \frac{\partial\mathcal{L}}{\partial\Phi}, \tag{13}$$

where on the LHS the divergence of the variational derivative with respect to $\partial_i\Phi$ is being taken. Note that $\mathcal{L}_{\rm NG}$ as defined here does not have the dimensions of a Lagrangian density, which will also be the case, hereafter, each time a subscript will be used.

Modifying gravity at the classical level means modifying $\mathcal{L}_{\rm NG}$ hereabove. In order to arrange for the differential operator in the deep-MOND regime, $\nabla\cdot\left(|\nabla\Phi|\,\nabla\Phi\right)$, which has two powers of $\nabla\Phi$ inside the divergence, the original Lagrangian must have three, namely $|\nabla\Phi|^3$. Written in terms of the *scalar* $\nabla\Phi\cdot\nabla\Phi$, that would then be $\left(\nabla\Phi\cdot\nabla\Phi\right)^{3/2}$. This is the origin of the non-canonical kinetic term with the peculiar power $3/2$, giving its name to this classical modified gravity theory: as it is not quadratic (or a-quadratic), it is called AQUAL, as an acronym of 'A-quadratic Lagrangian'. Now, letting $Y = (\nabla\Phi/a_0)\cdot(\nabla\Phi/a_0)$, in order to find an appropriate Lagrangian in the general case with $\nabla\cdot\left(\mu(|\nabla\Phi|/a_0)\,\nabla\Phi\right)$, one must find a function $\mathcal{F}$ such that $\mathcal{F}'(Y) = \mu(\sqrt{Y})$, as that function will be found differentiated inside the divergence operator, in other words the integral, $\mathcal{F}(Y) = \int \mathrm{d}Y\,\mu(\sqrt{Y})$. The gravitational Lagrangian density of AQUAL replacing $\mathcal{L}_{\rm NG}$ is

$$\mathcal{L}_{\rm AQUAL} = a_0^2\mathcal{F}((\nabla\Phi)^2/a_0^2), \tag{14}$$

where

$$\mathcal{F}(Y) \to Y \text{ for } Y \gg 1 \text{ and } \mathcal{F}(Y) \to \frac{2}{3}Y^{3/2} \text{for } Y \ll 1. \tag{15}$$

Source :[13]Famaey, B., & Durakovic, A. (2025). arXiv.
https://doi.org/10.48550/arXiv.2501.17006