GPS in Space: Relativistic Distortions in XNAV Near a Black Hole

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Abstract

Special relativity describes the relative nature of space and time, while general relativity explains gravity not as a force, but as the curvature of spacetime caused by mass and energy. Spacecraft navigation systems such as the Global Positioning System (GPS) and the new X-ray Pulsar Navigation (XNAV) concept both depend on precise timing to calculate position, meaning both are sensitive to relativistic effects. Near Earth, these effects are small and can be corrected as minor adjustments layered on top of an otherwise flat-space calculation. Near a black hole, however, that assumption no longer works. This paper investigates how gravitational redshift, Shapiro delay, and gravitational lensing would corrupt an XNAV system's ability to determine a spacecraft's position near a Schwarzschild black hole. Using Python-based simulations normalized to the Schwarzschild radius, we model three separate effects: the degradation of a received signal's frequency as a function of distance from the event horizon, the positional error introduced when Shapiro delay is left unaccounted for across a range of impact parameters, and the angular deflection of an incoming signal caused by gravitational lensing. Our results show that all three effects remain small and manageable at large distances, but increase sharply and nonlinearly once the signal's path nears the photon sphere, to the extent where an uncorrected system would mistake these relativistic distortions for genuine errors in the spacecraft's position. As for a lighter and more intuitive understanding of these results, we also generated a set of audio recordings that model how the pitch and speed of a transmitted signal would change with varying black hole mass and emitter distance. When put together, this paper seeks to show that deep space navigation must take relativistic corrections into serious consideration, and corrections would need to be built directly into the timing and position model from the start.

Introduction

Navigation in space is, at its core, an issue of time. To determine a spacecraft's location, the navigation system must compare the time at which a signal was emitted with the time at which it was received. Since electromagnetic signals travel at the speed of light, a difference in arrival time can be converted into a distance. In the Global Positioning System (GPS), receivers measure the travel time of radio signals transmitted by several satellites with atomic clocks. Those travel times then allow the receiver to calculate its distance from each satellite and solve for its three-dimensional position. However, this method only works if the system accounts for extremely small timing errors. Because light travels about thirty centimeters in one nanosecond, an error of only a few nanoseconds can noticeably change a receiver's calculated position.

GPS is one of the most famous applications of Einstein's theories of relativity [1][2]. The atomic clocks onboard GPS satellites are affected by two competing relativistic effects. First, because the satellites move rapidly relative to observers on Earth, special relativity causes their clocks to run slightly more slowly. Second, because the satellites orbit far above Earth's surface where gravity is weaker, general relativity causes their clocks to run more quickly than clocks on the ground. The gravitational effect is larger, so GPS satellite clocks would gain approximately 38 microseconds per day if the system did not correct them. While the difference may seem insignificant, it would cause GPS position errors to grow by several kilometers per day. Because of this, GPS engineers use a common coordinate-time system and continuously correct satellite clock rates and signal-propagation effects [3].

The need for a navigation system that does not depend entirely on Earth-based satellites becomes more important as spacecraft travel farther into the Solar System. A possible solution is X-ray pulsar navigation, usually called XNAV. Pulsars are rapidly rotating neutron stars that produce highly regular electromagnetic pulses. Some millisecond pulsars rotate hundreds of times per second and emit X-ray pulses with timing patterns stable enough to be used as celestial reference signals. Instead of receiving signals from an artificial GPS constellation, an XNAV spacecraft would observe several pulsars and compare the measured pulse arrival times with predicted arrival times from a pulsar timing model. Differences between the predicted and measured pulse phases can then be used to estimate the spacecraft's position and clock error. NASA's SEXTANT experiment demonstrated that pulsar observations can be used for autonomous, GPS-like spacecraft navigation [4].

Although XNAV and GPS use different signal sources, they share the same central principle: position is calculated from timing. This similarity also means that both systems must account for relativity. Near Earth, relativistic effects are important refinements to an otherwise nearly flat-space navigation problem. Near a black hole, however, relativistic effects would become much stronger and could no longer be treated as small adjustments. A spacecraft moving near a black hole would experience gravitational time dilation, meaning that its onboard clock would tick at a different rate from a distant reference clock. If the black hole is nonrotating and electrically neutral, its gravitational field can be described by the Schwarzschild metric. For a stationary clock at radial distance r, the relationship between its proper time τ and a distant coordinate time t is

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{r_s}{r}}$$

where

$$r_s = \frac{2GM}{c^2}$$

is the Schwarzschild radius of a black hole of mass M. As the spacecraft approaches the event horizon, r approaches r_s, and the relationship between local spacecraft time and distant coordinate time becomes increasingly different.

Clock-rate changes are not the only difficulty. The path of a pulsar signal would also be affected by curved spacetime. In ordinary GPS calculations, the signal path is close enough to a straight line that the range can be approximated as distance divided by the speed of light. Near a black hole, a pulsar pulse passing through the gravitational field experiences additional travel time, known as Shapiro delay. This delay can be written as

$$\Delta t_{delay} \approx \frac{2GM}{c^3}\ln(\frac{r_e+r_r+R}{r_e+r_r-R})$$

where r_e and r_r are the distances of the emitter and receiver from the gravitating object, and R is their coordinate separation. Near a black hole, especially when the black hole lies between the pulsar and spacecraft, the signal path may also bend through gravitational lensing. In extreme cases, multiple possible paths around the black hole could produce multiple images or delayed copies of the same pulsar pulse. A navigation system that assumes only one direct, straight-line signal path could mistake these delays for an error in spacecraft position.

This paper investigates how an XNAV system should correct pulsar timing near a Schwarzschild black hole. It focuses on four connected effects: gravitational time dilation of the spacecraft clock, special-relativistic time dilation caused by spacecraft motion, Shapiro delay along the pulsar-signal path, and gravitational bending of the signal trajectory. The paper compares these effects with the relativistic corrections already used in GPS, then considers how the XNAV timing and position equations must be modified in a strong gravitational field. The central argument is that relativity would not be a small correction added after calculating a spacecraft's position. Instead, near a black hole, relativistic spacetime geometry would need to be included directly in the navigation model that converts pulsar pulse-arrival times into the spacecraft's estimated (x), (y), and (z) coordinates.

Background on Special Relativity

At the end of the nineteenth century, physicists faced a major contradiction between established theories of light and motion. Maxwell's electromagnetic equations [5] predicted that all electromagnetic waves traveled at 3×10^8 m/s in vacuum, and light was originally assumed to travel via the medium "aether". However, the Michelson-Morley experiment [6] discovered that the speed of light within a vacuum is independent of the Earth and Sun, thereby disproving the idea that aether is necessary for light to travel. This resulted in a conflict with traditional Galilean ideas of relative motion, which suggested that velocities should be added or subtracted depending on the observer's reference frame. To resolve this contradiction, Albert Einstein proposed his theory of Special Relativity [1], founded on the principle that the laws of physics are the same in all inertial reference frames and that the speed of light in a vacuum is constant for all observers, regardless of their motion.

One of the most important and well-known consequences of special relativity is time dilation, the phenomenon in which time passes at different rates for observers in relative motion. Meaning, an observer watching a moving clock will find it to tick more slowly than a clock at rest relative to them. This is described by the time dilation equation

$$t = \gamma\tau,$$

where t is the dilated time measured by the stationary observer, τ is the proper time measured in the moving object's own frame of reference, and γ is the Lorentz factor [7]. As an object's velocity approaches the speed of light, the value of γ increases significantly, causing the observed passage of time to slow dramatically. Special relativity also predicts length contraction, in which the length of an object measured along the direction of its motion appears shorter to a stationary observer.

To relate the measurements made in different inertial reference frames, special relativity uses the Lorentz transformations:

$$t' = \frac{t - v(\frac{x}{c^2})}{\sqrt{1 - \frac{v^2}{c^2}}},\ x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}},\ y' = y,\ z' = z$$

These equations describe how the space-time coordinates of a specific event are transformed between observers in different reference frames. Unlike the Galilean transformations of classical mechanics, the Lorentz transformations account for the constancy of the speed of light. The transformations show how space and time are interconnected via the four-dimensional structure known as spacetime. Because of this, observers in different frames may disagree on the measured position, time, and duration of an event while the laws of physics remain the same for all reference frames.

Background on Spacetime Around Black Holes

The description of varying observations for a single event is further extended by general relativity through the explanation of gravity not as a conventional force, but as a curvature of spacetime caused by mass and energy. The Schwarzschild metric [8],

$$ds^2 = -(1 - \frac{2GM}{rc^2})c^2 dt^2 + (1 - \frac{2GM}{rc^2})^{-1} dr^2 + d\theta^2$$

is a solution to Einstein's field equations [2] that describes the geometry of spacetime around a spherical, non-rotating mass (or a Schwarzschild black hole) and is used to compute distances in that curved spacetime. Here, *M* is the mass of the object, *r* is the distance from its center, *G* is the gravitational constant, and *c* is the speed of light. The quantity ds^2 represents the spacetime interval, or the distance between two nearby events in curved spacetime. The metric shows that with the presence of such a large mass, measured distances and passages of time will change for nearby observers.

A significant consequence of the Schwarzschild metric is the Schwarzschild radius, given as

$$r_s = \frac{2GM}{c^2}.$$

For an object within this radius, the boundary at r_s becomes the event horizon, or the defining boundary of a black hole. Within this radius, nothing can escape—not even light. If an object fell inward, crossing this boundary, no signal or light emitted within it can travel outwards to a distant observer. Therefore, once information passes inside the event horizon, it cannot escape the black hole's gravitational influence.

Outside the event horizon, the Schwarzschild metric also provides the gravitational redshift factor,

$$\alpha = \sqrt{1 - \frac{r_s}{r}}$$

where $0 < \alpha < 1$. This factor measures how strongly gravity affects an observer a distance r from the black hole. The passage of time near the black hole is related to the time measured by a distant observer: as α becomes smaller, time passes more slowly for the nearby observer relative to the distant one. As r approaches the Schwarzschild radius, α approaches zero, showing the extreme gravitational time dilation that occurs near the event horizon.

Background on GPS and Relativistic Navigation

The GPS determines a receiver's location by measuring the travel times of radio signals from satellites. Each GPS satellite carries an atomic clock and broadcasts the time at which its signal was transmitted. A receiver then compares that transmission time with its reception time and converts the difference into a distance.

For satellite j, the simplest range equation is

$$c(t - t_j) = r - r_j = R_j$$

where c is the speed of light, t_j is the transmission time, t is the reception time, r is the position of the receiver, and r_j is the position of the satellite. When placed in three dimensions,

$$R_j = \sqrt{(x - x_j)^2 + (y - y_j)^2 + (z - z_j)^2}.$$

The receiver must find its unknown position (x, y, z) and determine its own clock error, Δt_r, because ordinary receivers don't contain atomic clocks. As a result, GPS needs signals from at least four satellites to solve for the four spacetime variables, $x, y, z,$ and Δt_r.

GPS also depends on extremely accurate timekeeping. A timing error changes the calculated distance by

$$\Delta R = c\Delta t.$$

Therefore, a one-nanosecond error could cause a thirty-centimeter range error. This is precisely why GPS must account for relativity; and because the GPS satellite clocks move so quickly, special relativity makes them run slightly slower, given by the formula:

$$\frac{d\tau}{dt} \approx 1 - \frac{v^2}{2c^2}.$$

At the same time, satellites orbit far above Earth, where gravity is weaker. As a result, general relativity makes their clocks run faster:

$$\frac{d\tau}{dt} \approx 1 - \frac{GM}{rc^2}.$$

The gravitational effect tops the motion effect. Together, GPS satellite clocks would run around 38.5 microseconds faster per day than clocks on Earth if they weren't corrected. GPS compensates for this clock difference before using satellite signals to calculate position.

GPS also corrects for effects such as Earth's rotation, changes in satellite orbit, and small delays as a signal moves through Earth's atmosphere and gravitational field. This principle is equally important for XNAV near a black hole, where relativistic effects would be much stronger and would directly affect the calculated spacecraft position. [3]

Background on the X-ray Pulsar Navigation (XNAV) System

X-ray pulsar navigation, or XNAV, is a way to find a spacecraft's position using X-ray pulses emitted by pulsars. Pulsars are rapidly rotating neutron stars whose pulses repeat at highly predictable rates. Like GPS, XNAV determines position from timing; however, it uses celestial sources instead of Earth-orbiting satellites for reference. The spacecraft records X-ray photons from several pulsars and compares their arrival times with a stored timing model for each pulsar [9].

A spacecraft does not receive perfectly separated pulses. Instead, it collects individual X-ray photon events, including both pulsar photons and background noise. The detected photon rate is given as

$$\lambda(t) = \alpha h(\varphi_{det}(t)) + \beta$$

where $h(\varphi_{det})$ is the known pulse-profile shape, α is the pulsed photon count rate, and β is the background count rate. The navigation system estimates the pulse phase, φ_{det}, by comparing the collected photon data with the known pulse profile.

For a stationary spacecraft, the detected pulse phase is approximately

$$\varphi_0 + F_0(t - t_0),$$

where φ_0 is the pulse phase at starting time t_0, and F_0 is the pulsar frequency. If the spacecraft moves toward or away from the pulsar, its motion changes the observed frequency through the Doppler effect. For motion directly along the pulsar-spacecraft line, we are given

$$f_0 = (1 + \frac{v}{c})F_0,$$

where v is the spacecraft velocity toward the pulsar and c is the speed of light.

A pulse arriving earlier or later than expected means that the spacecraft is not where the navigation model predicted. The approximate position error along a pulsar's direction is

$$c\frac{\Delta\varphi}{f}$$

where $\Delta\varphi$ is the pulse-phase error and f is the pulsar frequency. One pulsar only gives position information along its own line of sight, so several pulsars are needed to determine the full spacecraft position and clock error.

XNAV also requires a highly accurate onboard clock to record the arrival time of individual X-ray photons. Ashby and Howe describe finding pulse arrival times by comparing real-time observations with a known pulsar pulse profile [10]. The SEXTANT experiment later showed that XNAV can estimate spacecraft position, velocity, and time using millisecond pulsars [4].

Near a Schwarzschild black hole, this basic timing model would need additional corrections. Gravitational time dilation would change the spacecraft clock rate, while Shapiro delay and gravitational lensing would change the pulse travel time and path. Without those corrections, the XNAV system could mistake relativistic effects for errors in the spacecraft's x, y, and z coordinates.

Background on the Shapiro Delay

Shapiro delay is defined as the extra time taken by light or another electromagnetic signal when it travels through a gravitational field. It is a prediction of general relativity that was first proposed by Irwin Shapiro in 1964. In flat space, a signal's travel time is simply distance divided by the speed of light. Near a massive object, however, spacetime curvature causes the signal to arrive later than the flat-space prediction [11].

Using the Schwarzschild radius, we find that in a weak gravitational field, the speed of light can be written as

$$c(r) = c\sqrt{1 - \frac{2GM}{rc^2}}.$$

This describes the signal using a distant coordinate-time system, and the extra coordinate travel time becomes the Shapiro delay.

For a signal passing a mass of M, the one-way delay is

$$\Delta t = \frac{2GM}{c^3}\ln[\frac{x_E+r_E}{x_P+r_P}]$$

where r_E and r_P describe the distances of the receiver and emitter from the gravitating mass, respectively, while x_E and x_P describe their positions along the signal path. A more massive object would produce a larger delay, and the delay would increase when the signal passes closer to the object.

Background on Gravitational Lensing

Gravitational lensing occurs when gravity bends the path of light. In general relativity, light travels along curved paths called lightlike geodesics so that a massive object can change the apparent direction, brightness, and arrival time of a distant light source.

Near a Schwarzschild black hole, lensing can actually become very strong. Light passing close to the black hole could travel around it before reaching the observer. The photon sphere, where light can orbit the black hole in an unstable circular path, is located at

$$r = 3m = \frac{3GM}{c^2}.$$

Light rays that approach this radius can circle the black hole many times before reaching the observer. As a result, one pulsar can actually produce multiple images, with each one following a different path and arriving at a different time. [12]

Methods

To investigate how a black hole's gravity would disrupt a spacecraft navigation system in deep space, we designed and executed three separate simulations in Google Colab. These simulations plot out the exact timing, frequency, and directional errors that would happen when a navigation signal passes through warped or curved spacetime. To make the math work for any-sized black hole, we normalized our distances to the Schwarzschild radius, giving us a baseline where $r_s = 1$. We used standard Python libraries such as NumPy to handle the math and Matplotlib to plot the final graphs.

For the first simulation, we wanted to see how the signal's frequency drops as you get closer and closer to the black hole. When light makes its way out of a heavy gravitational well, it loses energy, which stretches its wavelength and drops its frequency, leading to gravitational redshift. We simulated a virtual spacecraft to travel from a safe distance of $10r_s$ down to an incredibly close distance of $1.05r_s$ right above the event horizon. By calculating the ratio of the received frequency compared to the original emitted frequency $\frac{v_{rec}}{v_{emit}}$, we could see exactly where the signal gets so weak that a ship's radio hardware would simply black out.

Next, we looked at the distance errors that would be caused by the Shapiro time delay. Normally, a GPS or navigation computer assumes space is completely flat and uses the basic formula $d = c \cdot t$. But, near a black hole, gravity stretches physical space and slows down time, making the signal arrive late. To simulate this, we placed a pulsar beacon and a spacecraft far away from each other at $100r_s$. Then, we calculated how much "fake distance" an uncorrected navigation computer would accidentally add to its position because of that time lag. We tested this across a range of impact parameters b/r_s—or how close the light beam skims past the black hole—moving from $20r_s$ to $2.5r_s$.

For our third simulation, we simulated the directional illusions that would be caused by gravitational lensing. Gravity curves the path of light, meaning navigation signals are forced to travel along bent lines instead of straight ones. Using the same approach from $20r_s$ down to $2.5r_s$, we calculated the exact deflection angle in degrees. This shows how much the black hole bends the incoming wave. Because a spacecraft's sensors blindly assume light always travels in a straight line, this bending tricks the computer into seeing the pulsar beacon in possibly several wrong spots in the sky at different times, ruining the geometric triangulation a spacecraft would need to find its coordinates.

Finally, for fun, we demonstrated the effect of gravitational redshift on audio; the phrase "I lawb kitty cats" was recorded using Voice Memos, converted to a WAV file, and uploaded to Google Colab. The audio was simulated to cross by a black hole of varying mass and varying distances from the event horizon. The audio recording was processed with the Librosa and NumPy libraries. The calculated redshift factor was applied to alter the signal's frequency, producing versions of the recording that model how its observed pitch would change at different distances and masses from a black hole.

Results and Discussion

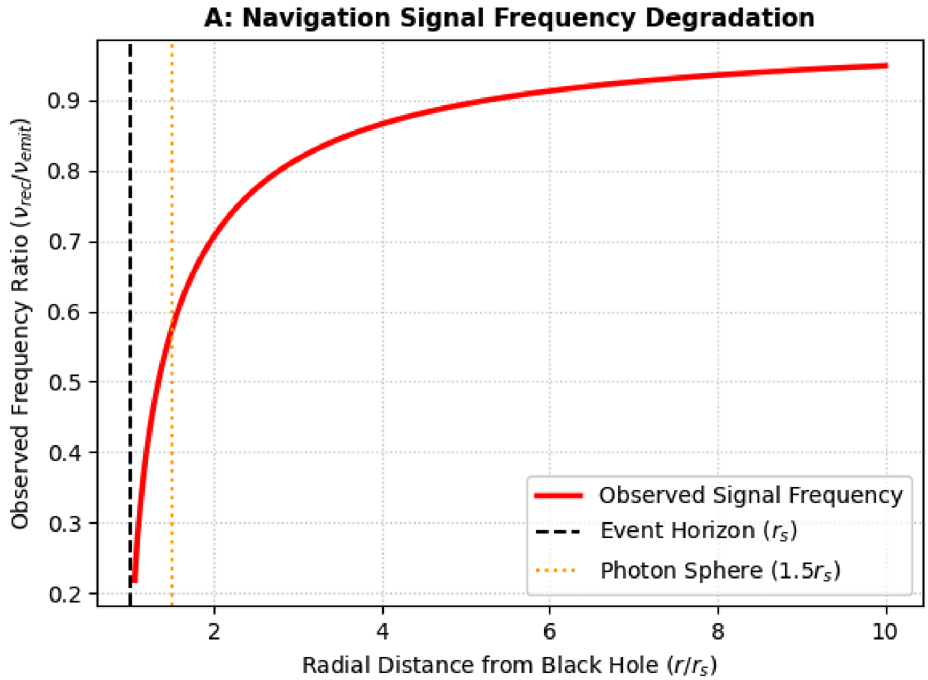

Figure 1: Signal Frequency Degradation.

Figure 1 shows how gravitational redshift affects and warps the frequency of the navigation signal as the receiver gets closer to the black hole. On the right end of the x-axis, the signal is a relatively safe distance away from the black hole. As a result, the frequency of the signal is not significantly altered, allowing for the ratio to remain close to a value of 1.0. However, as the signal is simulated to travel closer and closer to the black hole, there is a steep drop-off, indicating that the wave is being redshifted, and thus the frequency is decreasing at an exponential rate. As the radial distance approaches the event horizon, the frequency ratio approaches zero, signaling a zone where the hardware would blackout, unable to retrieve any information.

It is mathematically impossible to force a redshifted wave back to its original, emitted frequency. So, to counter such an obstacle would require adapting the physical hardware to "catch" the altered signals. As the spacecraft gets increasingly closer to the black hole, the computer onboard would have to use the gravitational redshift equation to calculate exactly how much the incoming waves are going to stretch them out. It would then have to adjust the tuning of its radio or X-ray antennas and shift the detection window down in order to "catch" the newly-lowered frequency signals. Furthermore, if the frequency shifted so drastically as to completely stretch into a different part of the light spectrum (e.g., shifting visible light into infrared), the system would have to automatically switch between sensors to keep the navigation data flowing and avoid a total blackout for communications.

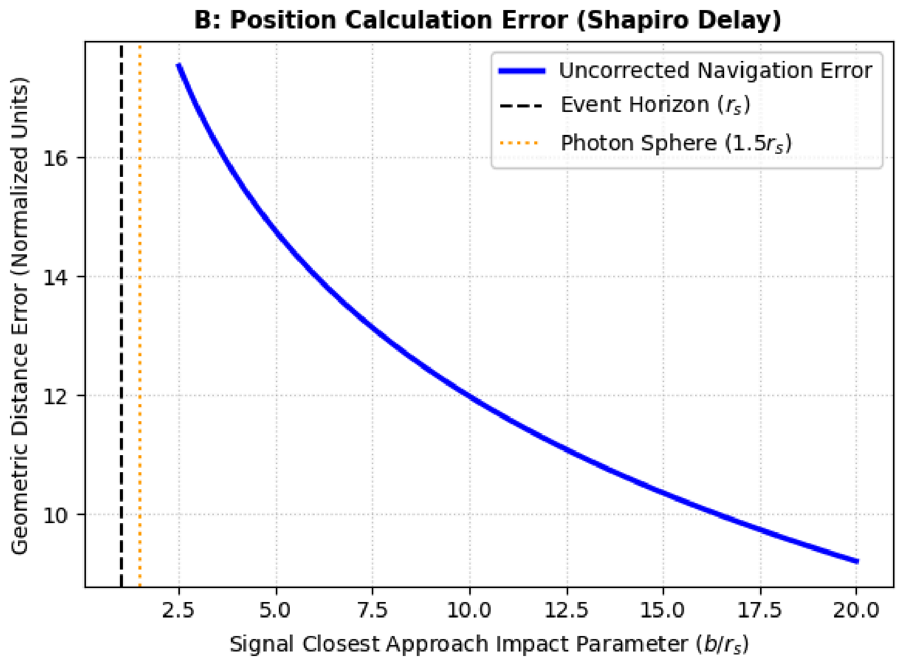

Figure 2: Position Calculation Error from Shapiro Time Delay

Figure 2 shows how critical spatial positioning errors could accumulate if the navigation system fails to account for general relativity. The x-axis tracks the signal's closest approach distance, or the impact parameter, as it passes by the black hole, moving from $20r_s$ down to $2.5r_s$. The blue curve shows that when a signal passes far from the black hole, spacetime curvature is weak, and flat-space positioning math will remain relatively accurate. However, as the signal's path approaches the black hole, time dilation and the stretching of spacetime combine to delay the signal's arrival time. And because navigation systems determine distance through $d = c \cdot t$, even a millisecond of general relativistic lag would've caused the onboard computers to miscalculate its coordinates by thousands of kilometers, resulting in the sharp, non-linear vertical spike on the left.

To fix such massive distance errors, the spacecraft's computer would essentially have to do the math backwards to figure out how late the signal was. Since the computer knows its approximate location and the location of the black hole, it can then calculate the signal's closest approach point, b. It would then plug that distance into the Schwarzschild metric to estimate exactly how many milliseconds the gravity well slowed down the signal. Once it knows that time lag, the computer would manually subtract it from the signal's arrival time before calculating its position. By taking away that "fake" extra travel time, the $d = c \cdot t$ formula would then give the ship its true, accurate distance from the beacon.

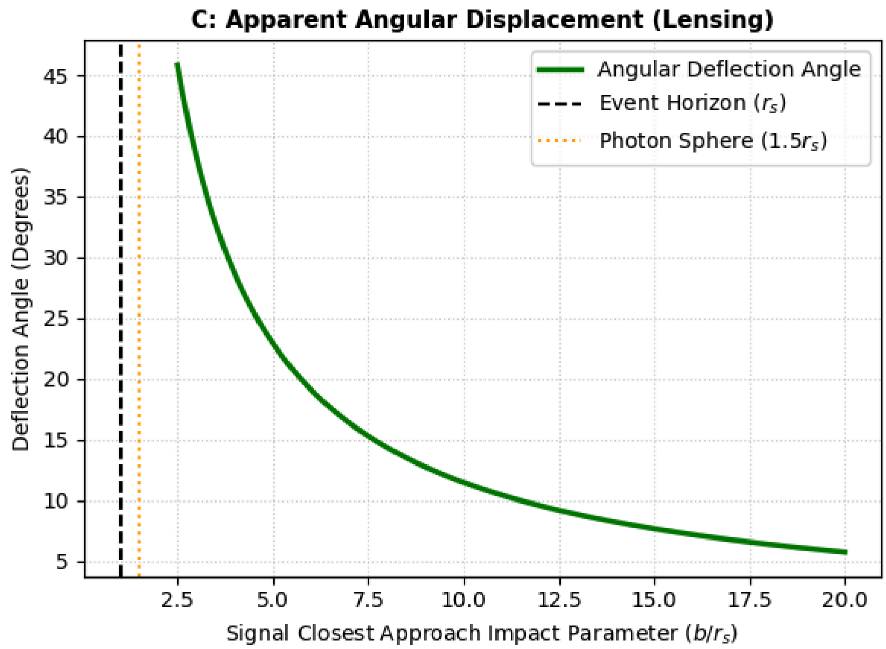

Figure 3: Apparent Angular Displacement Due to Gravitational Lensing

Figure 3 shows the severe directional errors that warp a spacecraft's navigation grid due to the gravitational bending of light. The horizontal axis measures the signal's closest approach distance, tracking how close the incoming wave skims past the black hole. The green curve shows that for signals passing far away from the black hole, the deflection angle remains virtually zero degrees, meaning light travels in its expected straight line. However, as the signal trajectory inches closer to the black hole, Einstein's warped spacetime forces the electromagnetic waves to follow heavily curved paths known as null geodesics. This causes the deflection angle to spike exponentially as it nears the photon sphere of $1.5r_s$. Because a navigation computer will blindly assume that light always travels in a straight line, this bending tricks the spacecraft's directional sensors into seeing the pulsar beacon in the completely wrong spot in the sky, ruining the geometric triangulation lines needed to accurately calculate the spacecraft's spatial coordinates.

Fixing this directional illusion from the gravitational lensing would require the navigation computer to unbend the incoming light waves. When a curved light path hits the spacecraft's sensors, the pulsar looks like it's shifted or jumping around in the sky. To rectify this, the computer would run a built-in ray-tracing algorithm. It takes the warped angle where the signal looks to be coming from, calculates the mass and distance of the black hole, and uses the deflection equation in reverse to trace the bent light path back to its straight-line origin. This mathematical correction gives the navigation system the true coordinates of the pulsar, allowing its geometric triangulation vectors to align correctly.

Additionally, we created [audio recordings] that reflect how waves may be affected when in the presence of black holes at varying distances and masses, demonstrating how gravitational redshift changes in pitch and playback speed. When a signal is emitted close to a black hole, it must travel outward through a strong gravitational field to reach a more distant observer. During this process, its observed frequency decreases. In the recordings with varying distances from the black hole, this frequency decrease is heard as a lower pitch and slower speech. The effect is strongest when the source is nearest to the event horizon, where spacetime is most strongly curved. As the observer is placed farther from the black hole, the signal is increasingly redshifted relative to its frequency near the black hole.

The recordings with varying black hole mass show that mass affects the strength of gravitational redshift. At a fixed distance, a more massive black hole has a larger Schwarzschild radius and produces a stronger gravitational field. Consequently, the emitted audio experiences a greater decrease in observed frequency, making the voice sound lower and slower. In contrast, a smaller black hole produces a weaker redshift at the same distance, so the observed audio remains closer to its original pitch and speed. These results provide an audible representation of the way gravity changes the frequency of electromagnetic waves and the rate at which time passes.

Conclusion

This paper aimed to examine how gravitational redshift, Shapiro delay, and gravitational lensing would disrupt an XNAV system operating near a Schwarzschild black hole, and to what extent these effects could realistically be corrected for. Taking into consideration the principles of special and general relativity, we built three simulations: one to model how a signal's frequency decays as it climbs out of the black hole's gravitational well, another to model how uncorrected Shapiro delay translates into large positional errors as a signal's closest approach nears the photon sphere, and finally, one to model how gravitational lensing bends a signal's path enough to distort the geometric triangulation a spacecraft would depend on to find its own coordinates. Across all three simulations, we saw a similar pattern of how relativistic effects stayed small and easily correctable at large distances from the black hole, but escalated dramatically and nonlinearly as the signal's path approached the photon sphere. This supports our central argument that relativity cannot be treated as a minor, after-the-fact correction near a black hole the way it can be for the GPS near Earth. Rather, it has to be built directly into the navigational model itself, well before any position is calculated. Along with the three simulations, we also created a set of audio recordings meant to give a more tangible, audible sense of how gravitational redshift alters a transmitted signal at different black hole masses and different distances from the event horizon. Looking forward, this model could be extended in a few interesting ways, such as allowing the emitter to move rather than staying stationary, replacing the Schwarzschild black hole with a rotating Kerr black hole, or perhaps introducing a second black hole to study how the effects would compound.

References

[1] Einstein, A. (1952). On the electrodynamics of moving bodies (W. Perrett & G. B. Jeffery, Trans.). In *The principle of relativity* (pp. 35–65). Dover Publications. (Original work published 1905)

[2] Einstein, A. (1952). The foundation of the general theory of relativity (W. Perrett & G. B. Jeffery, Trans.). In *The principle of relativity* (pp. 111–164). Dover Publications. (Original work published 1916)

[3] Ashby, N. (2003). Relativity in the Global Positioning System. *Living Reviews in Relativity, 6*, Article 1. https://doi.org/10.12942/lrr-2003-1

[4] Winternitz, L. M. B., Mitchell, J. W., & Hassouneh, M. A. (2018). SEXTANT X-ray pulsar navigation demonstration: Additional on-orbit results. *Proceedings of SpaceOps 2018: Conference on Space Operations*. https://ntrs.nasa.gov/api/citations/20180003358/downloads/20180003358.pdf

[5] Maxwell, J. C. (1996). *A dynamical theory of the electromagnetic field*. Wipf and Stock Publishers.

[6] Michelson, A. A., & Morley, E. W. (1887). On the relative motion of the Earth and the luminiferous ether. *American Journal of Science, 3*(203).

[7] Annamalai, C. (2023). *Lorentz factor and time dilation on the special theory of relativity* [Preprint]. TechRxiv. https://doi.org/10.36227/techrxiv.24297325.v1

[8] Schwarzschild, K. (1999). On the gravitational field of a mass point according to Einstein's theory. https://arxiv.org/abs/physics/9905030

[9] Wang, Y., Zheng, W., & Zhang, S. (2023). Review of X-ray pulsar spacecraft autonomous navigation. *Chinese Journal of Aeronautics, 36*(10), 44–63. https://doi.org/10.1016/j.cja.2023.03.002

[10] Ashby, N., & Howe, D. A. (2006). Relativity and timing in X-ray pulsar navigation. *Proceedings of the 2006 IEEE International Frequency Control Symposium and Exposition*. https://tsapps.nist.gov/publication/get_pdf.cfm?pub_id=50396

[11] Pössel, M. (2020). The Shapiro time delay and the equivalence principle (arXiv:2001.00229). *arXiv*. https://arxiv.org/pdf/2001.00229

[12] Perlick, V. (2010). Gravitational lensing from a spacetime perspective. *Living Reviews in Relativity* (arXiv:1010.3416). https://arxiv.org/pdf/1010.3416